International Journal of Engineering Innovation & Research
Volume 3, Issue 2, ISSN: 2277 – 5668

HLSRGM Based SPRT: MLE

Dr. R. Satya Prasad
Associate Professor,
Deptt. of Computer Science & Engg.,
Acharya Nagrjuna University, Nagarjuna Nagar, Guntur, Andhra Pradesh, India
Email: profrsp@gmail.com

S. Janardhana Rao
Team Lead, JKC, APSFKNW
Email: ieg. janardhan@gmail.com

G. Krishna Mohan
Reader, Deptt. of Computer Science,
P. B. Siddhartha College, Vijayawada, Andhra Pradesh, India
Email: km_mm_2000@yahoo.com

Abstract **– Sequential Analysis of Statistical science could be adopted in order to decide upon the reliability / unreliability of the developed software very quickly. The procedure adopted for this is, Sequential Probability Ratio Test (SPRT). It is designed for continuous monitoring. The likelihood based SPRT proposed by Wald is very general and it can be used for many different probability distributions. The parameters are estimated using Maximum Likelihood Estimation (MLE). In the present paper, the HLSRGM (Half Logistic Software Reliability Growth model) is used on five sets of existing software reliability data and analyzed the results.**

Keywords **– HLSRGM, MLE, Decision Lines, Software Testing, Software Failure Data.**

I. Introduction

Wald's procedure is particularly relevant if the data is collected sequentially. Sequential Analysis is different from Classical Hypothesis Testing were the number of cases tested or collected is fixed at the beginning of the experiment. In Classical Hypothesis Testing the data collection is executed without analysis and consideration of the data. After all data is collected the analysis is done and conclusions are drawn. However, in Sequential Analysis every case is analyzed directly after being collected, the data collected upto that moment is then compared with certain threshold values, incorporating the new information obtained from the freshly collected case. This approach allows one to draw conclusions during the data collection, and a final conclusion can possibly be reached at a much earlier stage as is the case in Classical Hypothesis Testing. The advantages of Sequential Analysis are easy to see. As data collection can be terminated after fewer cases and decisions taken earlier, the savings in terms of human life and misery, and financial savings, might be considerable.

In the analysis of software failure data we often deal with either Time Between Failures or failure count in a given time interval. If it is further assumed that the average number of recorded failures in a given time interval is directly proportional to the length of the interval and the random number of failure occurrences in the interval is explained by a Poisson process then we know that the probability equation of the stochastic process representing the failure occurrences is given by a Homogeneous Poisson Process with the expression

$$P\left[N(t)=n\right]=\frac{e^{-\lambda t}(\lambda t)^{n}}{n!} \qquad (1.1)$$

Stieber (1997) observes that if classical testing strategies are used, the application of software reliability growth models may be difficult and reliability predictions can be misleading. However, he observes that statistical methods can be successfully applied to the failure data. He demonstrated his observation by applying the well-known sequential probability ratio test (SPRT) of Wald (1947) for a software failure data to detect unreliable software components and compare the reliability of different software versions. In this paper we consider popular SRGM HLSRGM and adopt the principle of Stieber (1997) in detecting unreliable software components in order to accept or reject the developed software. The theory proposed by Stieber (1997) is presented in Section 2 for a ready reference. Extension of this theory to the SRGM – HLSRGM is presented in Section 3. Application of the decision rule to detect unreliable software with respect to the proposed SRGM is given in Section 4. Analysis of the application of the SPRT on five data sets and conclusions drawn are given in Section 5 and 6 respectively.

II. Wald's Sequential Test for a Poisson Process

The sequential probability ratio test (SPRT) was developed by A.Wald at Columbia University in 1943. Due to its usefulness in development work on military and naval equipment it was classified as 'Restricted' by the Espionage Act (Wald, 1947). A big advantage of sequential tests is that they require fewer observations (time) on the average than fixed sample size tests. SPRTs are widely used for statistical quality control in manufacturing processes. An SPRT for homogeneous Poisson processes is described below.

Let $\{N(t), t \geq 0\}$ be a homogeneous Poisson process with rate 'λ'. In our case, N(t)= number of failures up to time 't' and 'λ' is the failure rate (failures per unit time). Suppose that we put a system on test (for example a software system, where testing is done according to a usage profile and no faults are corrected) and that we want to estimate its failure rate 'λ'. We can not expect to estimate 'λ' precisely. But we want to reject the system with a high probability if our data suggest that the failure rate is larger than λ_1 and accept it with a high probability,

International Journal of Engineering Innovation & Research
Volume 3, Issue 2, ISSN: 2277 – 5668

if it's smaller than λ_0. As always with statistical tests, there is some risk to get the wrong answers. So we have to specify two (small) numbers 'α' and 'β', where 'α' is the probability of falsely rejecting the system. That is rejecting the system even if $\lambda \le \lambda_0$. This is the "producer's" risk. β is the probability of falsely accepting the system .That is accepting the system even if $\lambda \ge \lambda_1$. This is the "consumer's" risk. With specified choices of λ_0 and λ1 such that $0 < \lambda_0 < \lambda_1$, the probability of finding N(t) failures in the time span (0,t) with λ1, λ_0 as the failure rates are respectively given by

$$Q_1 = \frac{e^{-\lambda_1 t}\left[\lambda_1 t\right]^{N(t)}}{N(t)!} \quad (2.1)$$

$$Q_0 = \frac{e^{-\lambda_0 t}\left[\lambda_0 t\right]^{N(t)}}{N(t)!} \quad (2.2)$$

The ratio $\frac{Q_1}{Q_0}$ at any time 't' is considered as a measure of deciding the truth towards λ_0 or λ_1, given a sequence of time instants say $t_1 < t_2 < t_3 < \ldots\ldots < t_K$ and the corresponding realizations $N(t_1), N(t_2), \ldots\ldots N(t_K)$ of N(t). Simplification of $\frac{Q_1}{Q_0}$ gives

$$\frac{Q_1}{Q_0} = \exp(\lambda_0 - \lambda_1)t + \left(\frac{\lambda_1}{\lambda_0}\right)^{N(t)}$$

The decision rule of SPRT is to decide in favor of λ_1, in favor of λ_0 or to continue by observing the number of failures at a later time than 't' according as $\frac{Q_1}{Q_0}$ is greater than or equal to a constant say A, less than or equal to a constant say B or in between the constants A and B. That is, we decide the given software product as unreliable, reliable or continue the test process with one more observation in failure data, according as

$$\frac{Q_1}{Q_0} \ge A \quad (2.3)$$

$$\frac{Q_1}{Q_0} \le B \quad (2.4)$$

$$B < \frac{Q_1}{Q_0} < A \quad (2.5)$$

The approximate values of the constants A and B are taken as $A \cong \frac{1-\beta}{\alpha}$, $B \cong \frac{\beta}{1-\alpha}$

Where 'α' and 'β' are the risk probabilities as defined earlier. A simplified version of the above decision processes is to reject the system as unreliable if N(t) falls for the first time above the line $N_U(t) = a.t + b_2$ (2.6)

To accept the system to be reliable if N(t) falls for the first time below the line

$$N_L(t) = a.t - b_1 \quad (2.7)$$

To continue the test with one more observation on (t, N(t)) as the random graph of [t, N(t)] is between the two linear boundaries given by equations (2.6) and (2.7) where

$$a = \frac{\lambda_1 - \lambda_0}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (2.8)$$

$$b_1 = \frac{\log\left[\frac{1-\alpha}{\beta}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (2.9)$$

$$b_2 = \frac{\log\left[\frac{1-\beta}{\alpha}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (2.10)$$

The parameters α, β, λ_0 and λ_1 can be chosen in several ways. One way suggested by Stieber (1997) is

$$\lambda_0 = \frac{\lambda.\log(q)}{q-1}, \lambda_1 = q\frac{\lambda.\log(q)}{q-1}, \text{ where } q = \frac{\lambda_1}{\lambda_0}$$

If λ_0 and λ_1 are chosen in this way, the slope of NU (t) and NL (t) equals λ. The other two ways of choosing λ_0 and λ_1 are from past projects and from part of the data to compare the reliability of different functional areas.

III. HLSRGM

One simple class of finite failure NHPP model is the HLSRGM, assuming that the failure intensity is proportional to the number of faults remaining in the software describing an exponential failure curve. It has two parameters. Where, 'a' is the expected total number of faults in the code and 'b' is the shape factor defined as, the rate at which the failure rate decreases. The cumulative distribution function of the model is: $F(t) = \frac{\left(1-e^{-bt}\right)}{\left(1+e^{-bt}\right)}$.

The expected number of faults at time 't' is denoted by

$$m(t) = \frac{a\left(1-e^{-bt}\right)}{\left(1+e^{-bt}\right)}, a > 0, b > 0, t \ge 0.$$

The ML estimation of parameters 'a' and 'b' for the considered model is explained in Satyaprasad et. al.,(2011).

IV. Sequential Test for SRGMS

In Section II, for the Poisson process we know that the expected value of N(t) = λt called the average number of failures experienced in time 't' .This is also called the mean value function of the Poisson process. On the other hand if we consider a Poisson process with a general function m(t) as its mean value function the probability equation of a such a process is

$$P[N(t)=Y]=\frac{[m(t)]^{y}}{y!}.e^{-m(t)}, y=0,1,2,----$$

Depending on the forms of m(t) we get various Poisson processes called NHPP.
We may write

$$Q_1=\frac{e^{-m_1(t)}.[m_1(t)]^{N(t)}}{N(t)!}$$

$$Q_0=\frac{e^{-m_0(t)}.[m_0(t)]^{N(t)}}{N(t)!}$$

Where, $m_1(t)$, $m_0(t)$ are values of the mean value function at specified sets of its parameters indicating reliable software and unreliable software respectively. Let P_0, P_1 be values of the NHPP at two specifications of b say b_0, b_1 where $(b_0 < b_1)$ respectively. It can be shown that for our models $m(t)$ at b_1 is greater than that at b_0. Symbolically $m_0(t) < m_1(t)$. Then the SPRT procedure is as follows:

Accept the system to be reliable if $\frac{Q_1}{Q_0} \le B$

$$\text{i.e., } \frac{e^{-m_1(t)}.[m_1(t)]^{N(t)}}{e^{-m_0(t)}.[m_0(t)]^{N(t)}} \le B$$

$$\text{i.e., } N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} \quad (4.1)$$

Decide the system to be unreliable and reject if $\frac{Q_1}{Q_0} \ge A$

$$\text{i.e., } N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} \quad (4.2)$$

Continue the test procedure as long as

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} \quad (4.3)$$

Substituting the appropriate expressions of the respective mean value function – m(t) of HLSRGM, we get the respective decision rules and are given in followings lines

Acceptance region:

$$N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right)+\left[\frac{2a\left(e^{-b_0t}-e^{-b_1t}\right)}{1+e^{-b_0t}+e^{-b_1t}+e^{-t(b_0-b_1)}}\right]}{\log\left[\left(\frac{1-e^{-b_1t}}{1+e^{-b_1t}}\right)\left(\frac{1+e^{-b_0t}}{1-e^{-b_0t}}\right)\right]} \quad (4.4)$$

Rejection region:

$$N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right)+\left[\frac{2a\left(e^{-b_0t}-e^{-b_1t}\right)}{1+e^{-b_0t}+e^{-b_1t}+e^{-t(b_0-b_1)}}\right]}{\log\left[\left(\frac{1-e^{-b_1t}}{1+e^{-b_1t}}\right)\left(\frac{1+e^{-b_0t}}{1-e^{-b_0t}}\right)\right]} \quad (4.5)$$

Continuation region:

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right)+\left[\frac{2a\left(e^{-b_0t}-e^{-b_1t}\right)}{1+e^{-b_0t}+e^{-b_1t}+e^{-t(b_0-b_1)}}\right]}{\log\left[\left(\frac{1-e^{-b_1t}}{1+e^{-b_1t}}\right)\left(\frac{1+e^{-b_0t}}{1-e^{-b_0t}}\right)\right]} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right)+\left[\frac{2a\left(e^{-b_0t}-e^{-b_1t}\right)}{1+e^{-b_0t}+e^{-b_1t}+e^{-t(b_0-b_1)}}\right]}{\log\left[\left(\frac{1-e^{-b_1t}}{1+e^{-b_1t}}\right)\left(\frac{1+e^{-b_0t}}{1-e^{-b_0t}}\right)\right]} \quad (4.6)$$

It may be noted that in the above model the decision rules are exclusively based on the strength of the sequential procedure (α,β) and the values of the respective mean value functions namely, $m_0(t)$, $m_1(t)$. If the mean value function is linear in 't' passing through origin, that is, m(t) = λt the decision rules become decision lines as described by Stieber (1997). In that sense equations (4.1), (4.2), (4.3) can be regarded as generalizations to the decision procedure of Stieber (1997). The applications of these results for live software failure data are presented with analysis in Section 5.

V. SPRT Analysis of Live Data Sets

The developed SPRT methodology is for a software failure data which is of the form [t, N(t)]. Where, N(t) is the failure number of software system or its sub system in 't' units of time. In this section we evaluate the decision rules based on the considered mean value function for Five different data sets of the above form, borrowed from (Xie, 2002), (Pham, 2006) and (LYU,1996). The procedure adopted in estimating the parameters is a MLE. Based on the estimates of the parameter 'b' in each mean value function, we have chosen the specifications of $b_0 = b-\delta$, $b_1 = b+\delta$ equidistant on either side of estimate of b obtained through a Data Set to apply SPRT such that b0 < b < b1. Assuming the value of $\delta = 0.001$, the choices are given in the following table.

Table 5.1: Estimates of a, b & Specifications of b_0, b_1 for Time domain

Data Set	Estimate of 'a'	Estimate of 'b'	b_0	b_1
XIE	31.524466	0.005006	0.004006	0.006006
NTDS	28.767254	0.011941	0.010941	0.012941
AT&T	23.655141	0.004878	0.003878	0.005878
IBM	18.610725	0.007537	0.006537	0.008537
LYU	27.139750	0.039468	0.038468	0.040468

Using the selected b_0, b_1 and subsequently the $m_0(t)$, $m_1(t)$ for the model, we calculated the decision rules given by Equations 4.4 and 4.5, sequentially at each 't' of the data sets taking the strength (α, β) as (0.05, 0.2).

These are presented for the model in Table 5.2. The following consolidated table reveals the iterations required to come to a decision about the software of each Data Set.

Table 5.2: SPRT analysis for 5 data sets of Time domain data

Data Set	T	N(t)	Acceptance region (≤)	Rejection Region (≥)	Decision
Xie	30.02	1	-1.727836	9.005988	Reject
	31.46	2	-1.636667	9.101065	
	53.93	3	-0.330788	10.491850	
	55.29	4	-0.258575	10.570651	
	58.72	5	-0.079792	10.766786	
	71.92	6	0.564557	11.487834	
	77.07	7	0.797663	11.755159	
	80.9	8	0.964552	11.949070	
	101.9	9	1.785280	12.942073	
	114.87	10	2.216340	13.500214	
	115.34	11	2.230922	13.519702	
	121.57	12	2.417539	13.773357	
	124.96	13	2.513939	13.907810	
	134.07	14	2.755478	14.257165	
	136.25	15	2.809568	14.338265	
NTDS	9	1	-7.843729	18.001922	Continue
	21	2	-6.233833	19.832794	
	32	3	-5.038984	21.387852	
	36	4	-4.666287	21.929605	
	43	5	-4.088848	22.852718	
	45	6	-3.940770	23.111367	
	50	7	-3.602309	23.749589	
	58	8	-3.151720	24.750483	
	63	9	-2.924613	25.366841	
	70	10	-2.674069	26.222722	
	71	11	-2.644525	26.344620	
	77	12	-2.499156	27.075715	
	78	13	-2.480164	27.197673	
	87	14	-2.374692	28.300704	
	91	15	-2.364775	28.796077	
	92	16	-2.365774	28.920608	
	95	17	-2.377026	29.296111	
	98	18	-2.400548	29.674830	
	104	19	-2.483874	30.443693	
	105	20	-2.502414	30.573511	
	116	21	-2.792649	32.039628	
	149	22	-4.593984	37.041995	
	156	23	-5.156011	38.261485	
	247	24	-19.279695	63.328885	
	249	25	-19.766714	64.156455	
	250	26	-20.013819	64.576280	
AT&T	5.5	1	-3.439108	6.975118	Reject
	7.33	2	-3.339245	7.075937	
	10.08	3	-3.191409	7.225722	
	80.97	4	-0.230303	10.451384	
	84.91	5	-0.109560	10.599110	
	99.89	6	0.311758	11.135113	
	103.36	7	0.401058	11.253734	
	113.32	8	0.640650	11.583310	
	124.71	9	0.885167	11.941429	
	144.59	10	1.240442	12.522872	
	152.4	11	1.356361	12.737542	
	167	12	1.539190	13.120313	
	178.41	13	1.652676	13.404253	
	197.35	14	1.787370	13.850204	
IBM	10	1	-5.174671	11.064752	Continue
	19	2	-4.635609	11.643718	
	32	3	-3.947484	12.433458	
	43	4	-3.444127	13.063760	
	58	5	-2.866785	13.875678	
	70	6	-2.489933	14.492294	
	88	7	-2.056376	15.374974	
	103	8	-1.807640	16.083028	
	125	9	-1.613573	17.098125	
	150	10	-1.620105	18.252469	
	169	11	-1.774906	19.156018	
	199	12	-2.266556	20.678868	
	231	13	-3.111709	22.507608	
	256	14	-4.000790	24.142446	
	296	15	-5.858138	27.263917	
LYU	0.5	1	-30.478801	54.971145	Continue
	1.7	2	-29.885268	55.623243	
	4.5	3	-28.701385	57.192835	
	7.2	4	-27.815080	58.783688	
	10	5	-27.148349	60.531188	
	13	6	-26.707619	62.534462	
	14.8	7	-26.574971	63.810749	
	15.7	8	-26.545029	64.471800	
	17.1	9	-26.546145	65.532413	
	20.6	10	-26.800138	68.371189	
	24	11	-27.387486	71.415866	
	25.2	12	-27.674819	72.565406	
	26.1	13	-27.917825	73.455030	
	27.8	14	-28.441489	75.202707	
	29.2	15	-28.936689	76.711268	
	31.9	16	-30.057132	79.809870	
	35.1	17	-31.673847	83.833409	
	37.6	18	-33.162317	87.268135	
	39.6	19	-34.500816	90.216291	
	44.1	20	-38.017728	97.568797	
	47.6	21	-41.269157	104.055170	
	52.8	22	-47.018099	115.122822	
	60	23	-57.045594	133.802814	
	70.7	24	-77.439058	170.827940	

From the Table 5.2, a decision of either to accept, reject the system or continue is reached much in advance of the last time instant of the data.

VI. Conclusion

The above consolidated table of HLSRGM as exemplified for five Data Sets indicates that the model is performing well in arriving at a decision. The model has given a decision of rejection for 2 Data Sets i.e. Xie and AT&T at 15th and 14th instances respectively and a decision of continue for 3 Data Sets i.e. NTDS, IBM and LYU. Therefore, we may conclude that, applying SPRT on data sets we can come to an early conclusion of reliability / unreliability of software.

References

[1] Lyu, M.R. (1996). "The hand book of software reliability engineering", McGrawHill & IEEE Computer Society press.
[2] Pham. H., (2006). "System software reliability", Springer.
[3] Satyaprasad (2007)."Half logistic Software reliability growth model "Ph.D Thesis of ANU, India.

[4] Satyaprasad, R., Ramchand, H. R. and Kantam, R. R. L., (April, 2011). "Software Reliability with SPC", International Journal of Computer Science & Emerging Technologies, Volume 2, Issue 2, pp[233-237].

[5] Stieber, H.A. (1997). "Statistical Quality Control: How To Detect Unreliable Software Components", Proceedings the 8th International Symposium on Software Reliability Engineering, 8-12.

[6] Wald. A., 1947. "Sequential Analysis", John Wiley and Son, Inc, New York.

[7] Xie, M., Goh. T.N., Ranjan.P. (2002). "Some effective control chart procedures for reliability monitoring" -Reliability engineering and System Safety 77 143 -150.

AUTHOR'S PROFILE

Dr. R. Satya Prasad

received Ph.D. degree in Computer Science in the faculty of Engineering in 2007 from Acharya Nagarjuna University, Andhra Pradesh. He received gold medal from Acharya Nagarjuna University for his outstanding performance in a first rank in Masters Degree. He is currently working as Associative Professor in the Department of Computer Science & Engineering, Acharya Nagarjuna University. His current research is focused on Software Engineering. He published 70 research papers in National & International Journals.

S. Janardhana Rao

presently working at JKC, Andhra Pradesh Society for Knowledge Networks/ Institute for Electronic Governance (APSFKNW) as a Team Lead for the Academic Program on Oracle Academy, Worked 5 years in Admin section for tracking and smooth functioning of JKC files online, Worked 2 years at Aditya Engineering College as JKC Coordinator. He is pursuing his Ph.D under the guidance of Dr. R.Satyaprasad at Acharya Nagarjuna University.

Dr. G. Krishna Mohan

working as a Reader and Head in the Department of Computer Science, P.B.Siddhartha College, Vijayawada. He obtained his M.C.A degree from Acharya Nagarjuna University, M.Tech. (CSE) from Jawaharlal Nehru Technological University, Kakinada, M.Phil from Madurai Kamaraj University and Ph.D(CSE) from Acharya Nagarjuna University. He qualified, AP State Level Eligibility Test. His research interests lies in Data Mining and Software Engineering. He published 23 research papers in various National and International journals.