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A Dynamical Model for Fracture Advance Using Stress Analysis and Empirical Crack-Growth Velocity Part 2. The Experimental Design

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***Abstract* – A new kind of mathematical models for simulation of crack growth was presented in Part 1. It conjugates dynamically stress analysis via FEM with an empirically determined crack-growth velocity. A main point in the development of the model was the relation between the theoretical requirements of data and the practical capability to obtain them. The model provided the main guidelines of the design of experiments. This approach to experimental design is basic to develop better experiments where the theory plays a fundamental role. It turned out that the productivity of the theoretical guidelines was so adequate that the information could be obtained with standard procedures and attention could be focused in the quality of the process. The approach was also important to establish the necessary procedures and data to validate the model. Once the data were obtained, the model was used to obtain an adequate simulation of the crack growth. This second part is focused on the interaction between mathematical model and design of experiments made possible by the capability of modern computers and thus conceived as a practical methodological contribution towards a new conception of engineering research.**

***Keywords* – Simulation, Fracture, Model, Crack Growth, Experimental Design.**

I. INTRODUCTION

1.1. Introduction. As mentioned in Part 1 [1], the original motivation for this research was a very concrete failure in the structure above the rear door of a bus. The physical situation is geometrically analog to simpler but representative experimental settings. The material was steel AISI 1020, a usual one with standardized characterization.

The only peculiarity was a thin stripe of welding almost parallel to the crack evolution. But besides that, and considering that an empirical estimation of the velocity of crack growth due to Paris and collaborators already exists [2], the numerical structure of a time-dependent Finite Element analysis allowed the construction of a dynamical numerical scheme combining crack advance velocity with stress analysis at every time step. The model is presented in 2.1.

This model was then used to establish the flow of data in the experimental process, giving guidelines that allowed better design of experiments and realization. The amount of theoretical considerations and experimental design issues made it necessary to write this second paper.

Basically, the model offers the possibility to simulate crack growth based on structured data from standard experiments. The crack growth interacts at each time step with the forces induced on the body, giving a dynamical Finite Element (FE) analysis of the stresses and their evolution. The model can be feed with data of other materials. The applied forces can also be changed as well as the geometry of the body and the initial and boundary conditions.

An important development along the research was the methodology used in the design of experiments, that is presented in the next section. The concrete design is given, together with some relevant data. Finally, complementary images to the ones presented in Part 1 are presented.

1.2. Principles of design of experiments based on a mathematical system (a model).

Certainly, the most basic requirement is a deep understanding of the model by the one who should design the experiments.

The model answers the fundamental question of every design: What are the objectives of the project as a whole? In the case here presented two general objectives direct the procedure:

I. Simulate the mechanical problem (fracture) as a dynamical unity of the action of the involved forces and the geometrical alteration of the specimen.

II. Once a flexible simulation tool has been developed and validated, it can be used to model other situations and to predict the evolution of stress by changing: a) the characterization of the material; b) the boundary conditions, specially the Neumann conditions representing the forces exerted on the material: and c) the geometrical configuration.

The next step is the determination of the required inputs to the model and its relation to realizable experimentation.

To establish the flow of information and the relation between the different data the model acts as a synthesis of the requirements and possibilities. For example, the model allows the determination of possible parallelisms indicating calculus that can be done independent of others while strictly sequential flow establishes an analog sequence of experimental measurements.

An analysis of the error and precision issues has to be done in order to guarantee the requirements of the objectives. This is done giving the precision of the data and the error generated along the flow of calculations.

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This aspect is not considered in the concrete case analyzed here.

Special attention is put on the issues of validation. In our case, for example, the differences between the calculated deformations of the region and the measured deformations of the real specimen are used to validate the process. These measurements have to be also integrated in an efficient way in the actual flow of experimental data.

At each step, the model provides an interpretation of the measured data, in the same sense that the concept of energy offers one interpretation of measured velocities and positions. This provides the experimenter with a theoretical insight that combines with his/her professional experience. For example, in the concrete case here considered the proposal of using a second degree polynomial to represent the shape of the deviated trajectory of the crack is based both in considerations related to the needs of modeling and efficient experimentation.

This non-linear curve is needed to calculate the empirical crack growth velocities in the direction of the crack growth. This velocity is defined as the tangent vector to the curve at a given point (x, y), The formula derived by Paris et al. [2], gives the velocity in the x-axe direction, that is then interpreted as the projection of the velocity on the curve. To obtain the direction of the tangent the derivate of the polynomial is used. The use of second-degree polynomials is also very well fitted to the numerical schemes.

This example shows how concrete the guidelines given by a model for experimental design can be.

II. Design of Experiments Guided by Fracture Simulation Model With Semi-Empirical Dynamics

The geometrical body Ω represents an experimental specimen of the same dimensions; see Fig.1, assumed homogeneous and isotropic and mechanically characterized by constants S and S. It is subject to periodic loads in an Instron. It is fixed in boundary $\partial\Omega_6$ and the forces are exerted in boundary $\partial\Omega_7$.

Once discretized the basic equation of linear elasticity using the FE method, and when the empirical crack-growth velocity has been used to approximate the second order term, the dynamical one depending on time, the resulting numerical scheme is presented in the next section.

2.1. The model, as presented in [1], has the form

$$\mathbf{M}u_{i+1} = F + \frac{v_{i+1}-v_i}{h}, \text{ on } \Omega_{i+1}$$

where M is the numerical matrix resulting from the FE discretization process, u_{i+1} is the vector of the unknown (approximate) values of the deformation function u at the nodes at iteration i+1; F is the corresponding numerical inhomogeneous tern that in our case (homogeneous) is equal to zero; the v_i are the empirically determined crack-growth velocities; the term $\frac{v_{i+1}-v_i}{h}$ gives a first order finite-differences approximation of the second derivate of u with respect to t, time, and with recursive Dirichlet boundary conditions

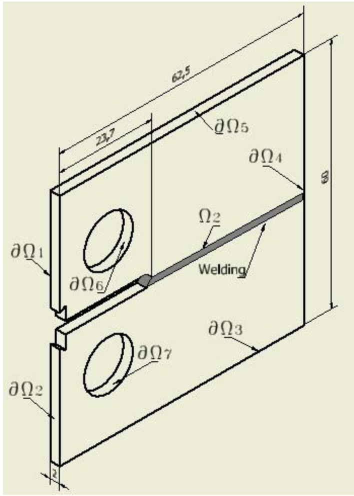

Fig.1. Body Ω with its boundaries ∂Ω and welding, dimensions in mm.

$u(p, t) = u_0$, in $\partial\Omega_i$, i =1, 2…

$u(p, t) = p_i$, $p_i = u_i(p)$ in $\partial\Omega_i$, i =1, 2…. p_i

and with Neumann conditions:

$$\frac{\partial u}{\partial n}(p,t) = K, \text{ in } \partial\Omega_i$$

Defined on the region Ω. See Fig. 1.

2.2. The system offers a simulation of the crack evolution and the dynamically corresponding stresses induced in the specimen. From the experimental point of view it defines the following sets of parameters and data.

A. The constants defining the mechanical characteristics of the material, specially the Lamé constants.

B. The initial geometry, including (or not) the area simulating the stripe of welding, and the Lamé constants of this inhomogeneity; also the determination of the areas with fixed Dirichlet conditions, those with known but not constant conditions on the deformations, those left free.

C. The Neumann boundary conditions, that is, how the applied forces are modeled.

2.2.1. In the analyzed case the data required to initialize the system are:

D.1.1. The constants and required data to characterize the material or materials.

D.1.2. The geometry of the region Ω discretized using the MEF; this set of data includes the FE parameters: type of FE, size and position of the FE, type of interpolation and weight functions, etc. It includes also the division of Ω in a finite number of sub-boundaries $\partial\Omega_i$, i = 1, 2, …, n, and the corresponding attribution of boundary conditions, free, Dirichlet, Neumann or mixed to each sub-boundary.

D.1.3. The concrete values of the Dirichlet conditions.

D.1.4. The concrete values of the Neumann conditions. It has to be indicated that the real Neuman conditions were given by periodic loads, but some considerations allows them to be taken as constant [1] (see Part 1).

2.2.2. Data that are given to the model at specific point of the flow from outside:

D.2.1. Data needed to calculate the crack growth velocity.
D.2.2. The aperture of the notch.
D.2.3. The modification in geometry given by the calculated crack advance.
2.2.3. Data that are given to the model at specific points of the flow from calculations in the previous iteration step are:
D.3.1. The (deformed) geometry in iteration n is given as initial geometry for iteration n+1.
D.3.2. Data from the real advance of the crack, for example the deviation of the crack from a rectilinear trajectory.
2.2.4. The data required to validate the process are:
D.4. Measurements of the geometry of the deformed specimen at pre-determined times.
2.3. Data flow in the model.
All data D.1.are needed to initialize the system, but only Data D.1.1. and the fixed values of the Dirichlet conditions are used along the whole process and has to be available.
Data D.2 and D.3.are feed to the system at each iteration.

After each iteration a validation step (see next section) is taken, an internal precision parameter ε> has to be established. If $\|\Omega - \Omega^{'}\| > \varepsilon$ the process is interrupted and a signal is given.

The data output is calculated and given as a series of images simulating the crack growth at constant time intervals.

2.4. The validation of the process is made comparing the experimentally measured consideration and the calculated one. To adequately interpret the differences between two geometrical configurations the following metric was employed:

Choose n points p_i evenly distributed in the boundary of Ω_0. Measure the experimental positions of the corresponding physical points after a fixed time t_0. Denote those new positions by p'_i. Calculate via the numerical procedure the new positions p_i'' of the same points after the (theoretical) time t_o. Then the following metric is defined between regular regions of three-dimensional space:

$$\|\Omega - \Omega^{'}\| = \frac{1}{n}\sum_{i=1}^{n}\|p_i^{'} - p_i^{''}\|$$

where $\|p_i^{'} - p_i^{''}\|$ is the Euclidian norm of the vector defined by p_i’ and p_i’’. It measures the difference between a configuration Ω and another Ω’. The metric showed adequate congruence between the experimental and the calculated configurations.

2.5. Experimental data flow.

The previous analysis allowed the determination of the experimental steps, its sequence and its precision.

All steps were standard procedures or not complex measurements, like the measure of the deformed configurations. The experimental sequence was strictly the same as determined in the previous section.

This determination allowed also to locate steps that can be automatized, specially the introduction of the data giving the deformed configuration. The measurements needed for validation can certainly be also obtained via photographies and image recognition.

III. THE EXPERIMENTS

3.1. Materials: The material used was obtained from the tubular profile of a steel AISI 1020. It was sectioned to obtain samples for chemical analysis and to machine the specimens for tensile and crack growth.

All specimens were obtained from the same plate and manufactured according to the recommendations of ASTM E08 – 00 [3] andASTM E647 -05 [4], by wire EDM process. Metallography and stress testing experiments were also performed with typical specimens.

3.2. Geometry of specimens for fracture mechanics.

The specimens used are type CT (Compact Tension), previous studies have tested this type specimens [5], whose geometry is shown in Fig. 1. Its dimensions are: width W = 50 mm and B = 2 mm thickness.

The tests were performed according to the standard ASTM E 647-05 at constant load and at a frequency of 10 Hz on the Instron 8801 servo-hydraulic. Crack length was measured by the standard methods. Opening displacement was measured with a strain gauge placed on the front face of the specimen.12 tests were performed for crack growth rate.

IV. RESULTS

The conjugation of stress analysis with the dynamic second order term approximated via a finite-differences scheme resulted on a complete simulation of the crack growth in dynamic interaction with the stresses induced in the body.

In Part 1 images representing the crack growth and corresponding stress data for the rectilinear and deviated case (due to the asymmetry in the loads) are presented [1]. In this paper the images corresponding to the case with a thin stripe of welding are shown.

V. CONCLUSIONS

Two main results are given in these two papers.

First of all a model allowing a complex but flexible simulation of a crack growth evolution along with the stresses is given for a concrete problem. This simulation can be made in diverse other situations.

The conceptual combination of stress evolution and empirical crack growth, using mainly the crack growth velocity in an efficient numerical scheme gives insight in the forces acting along the crack evolution.

On the other hand, this research offers a methodological approach to the new kind of engineering research where theory, represented by mathematical models incorporating empirical and elaborated data, interacts with experimental design and thus with the real practice of engineering.

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AUTHOR'S PROFILE

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He is currently a Ph. D. candidate in Mechanical Engineering at UNAM in México City. His current research interest include materials fatigue, fracture mechanics and kinematics of mechanisms. He has teaching experience of 10 years.

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Professor for pure and applied Mechanics at the Faculty of Engineering, UNAM. Doctor in applied Mathematics. Doctor in Philosophy of Science.Hismain focus of research and teaching is themathematical and computational modeling ofphysical systems using differential equations and theFinite Element Method for discretization. Pure and applied (toinformatics) logic is his other point of interest. He has been working atthe UNAM 27 years. Besides he has worked in many fields of applicationof mathematics.

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Received the B.S. degree inMetallurgical Engineeringfrom Faculty of Chemistry, UNAM (1993), Master of Science in Siderurgy from Technological Institute of Morelia (1996) and Ph. D. in Metallurgy and Materials Science from Polytechnic Institute Nacional (2007). Responsible Optical Microscopy Laboratory of the Mexican Institute of Transport (IMT), which has developed projects related to fracture mechanics and structural integrity. Industry experience of heat treatment of steel. Currently performs a Postdoctoral stay at the Cantabria University, Spain.

APPENDIX

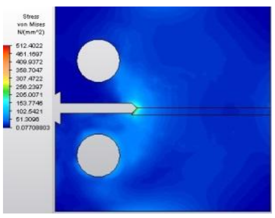

Fig.2. Set03 initial image.

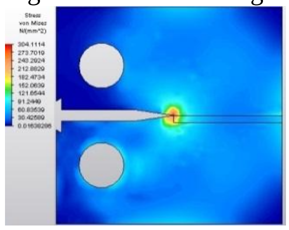

Fig.3. Set03 intermediate image.

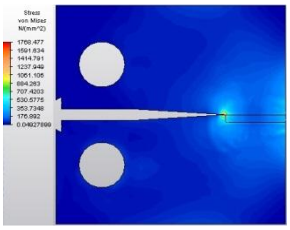

Fig.4. Set03 final image.

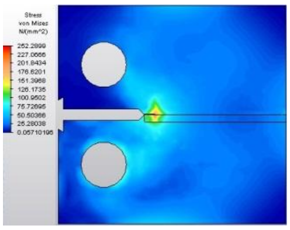

Fig.5. Set04 initial image.

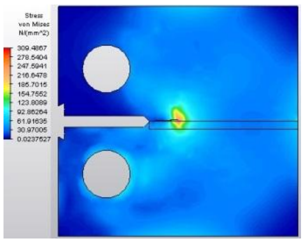

Fig. 6.Set04 intermediate image.

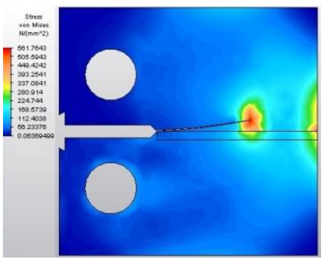

Fig.7. Set04 final image.

Fig.8. Specimenfractured.