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A Dynamical Model for Fracture Advance Using Stress Analysis and Empirical Crack-Growth Velocity Part 1. The Model

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***Abstract* – A new model is presented that allows a dynamical simulation of the interaction between crack advance and induced stresses in medium carbon steels under periodic and constant loads. It is based on the use of an empirical crack-growth velocity to approximate the second order term in the basic equation of linear elasticity. Once this is done, a Finite Element analysis of the stresses gives what is here called a semi-empirical numerical dynamic model of the crack evolution. The simulation requires a set of data obtained through experimentation that was by itself guided in its design by the model, in what can be considered as an example of an evolving new paradigm in engineering research. Several simulations of the crack evolution were obtained under different considerations for the Neumann boundary condition and with or without an inhomogeneity simulating a thin stripe of welding. The simulation allows the analysis of the crack evolution as the stresses in the body evolve. Due to the relevance of the experimental data and design, this research is presented in two parts, this first paper dealing with the model, its physical and mathematical aspects and giving examples of the results. The second one deals with the experimental aspects.**

***Keywords* – Crack, Fatigue, Finite Element, Scheme, Stress.**

I. Introduction

The original motivation for this work was a fatigue-generated crack in a localized part of the structure of a bus. The crack originated parallel to a thin stripe of welding. From this concrete problem a simple geometrical and mechanical setting was abstracted representing a broad class of design and stress situations for steel structures of medium size subject to periodical or constant loads. Basically, a crack is induced in a rectangular plate of the considered material. The evolution of the crack subject to non-symmetric periodic loads (one side of the specimen is fixed) is experimentally established. Even if the plate is relatively thin compared to the other spatial dimensions, the problem was considered three-dimensional [1], [2].

Specimens were designed, manufactured and then subjected to periodical loads in an Instron machine 8801 in order to generate cracks under two basic conditions: with or without a stripe of welding [3].

The experiments produced a considerable amount of precise data that together with the mechanical characterization of the specimens yielded information allowing the determination of the involved crack growth velocities, using the empirical formula developed by Paris et al. [4], as well as the needed parameters and geometric values for a Finite Element stress analysis.

An adequate methodology to combine both approaches in a novel and fruitful scheme is the main contribution of this work. Once the semi-empirical model was developed, the design of the experimental part was realized under strict guidance from it. We attempt to show here that an approach to experimental design based on a mathematical model (even if an important part of the simulation is made using semi-empirical developments) leads to better experiments because the design is pre-directed to the sought improvement of our knowledge and not only to an increase of information. This project is an example of an evolving new paradigm in engineering research to which we ascribe.

This first paper emphasizes the more theoretical developments and results and in the second one a careful explanation of the experiments and their design is given.

II. Modeling the Interaction Between Crack Advance and Induced Stresses in Medium Carbon Steels Under Periodic and Constant Loads

2.1. Physical setting, geometry and initial boundary conditions.

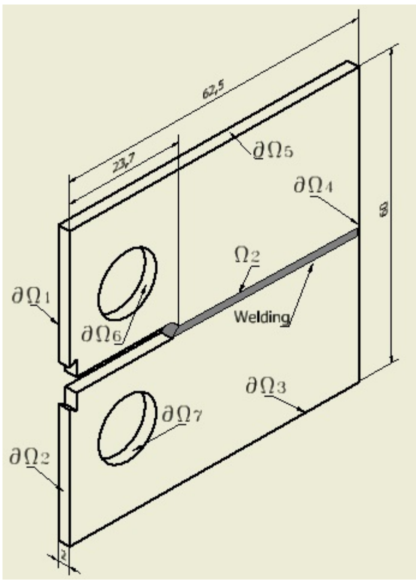

Fig.1. Body Ω with its boundaries ∂Ω and welding, dimensions in mm.

The considered geometrical body Ω corresponds to specimens made according to Norm E 647 – 05 [11]. They were made of steel AISI 1020. The dimensions are 60 by 62.5 by 2 mm (See Fig. 1). A notch of 23.7 mm was made at the center of one of the 60 mm sides. Thus, the body Ω is a subset of three-dimensional geometric space with the same dimensions.

The specimen was subject to periodic loads of 1.757 kN in an 8801 Instron until the crack reached 20 mm. See Part 2 for details and experimental design.

For the mathematical modeling the following considerations were made:

A. The problem is considered three-dimensional.

B. The employed FE software is based on linear elasticity. The whole physical setting is analyzed from the point of view of continuum mechanics, mainly as presented in [5]. The material is considered homogeneous and isotropic. It is characterized by the properties shown in Table 1.

Table 1: Mechanical properties of the material

Thickness	E(GPa)	Sy(MPa)	Su(MPa)
2.0	217	372	427

In the following step, a subset of Ω in the form of a prism of 2 by 2 by 38 mm located at Ω_2 (see Fig. 1) is assumed to model the change in properties due to the welding. This subset is in itself homogenous and isotropic but with constants Sy = 480 MPa y Su = 400 MPa, reflecting the differences between both materials. Continuity conditions were imposed at the boundaries.

C. Experimentally, the specimen was mounted on the Instron at two circular areas equidistant to the expected crack evolution. In one of these areas the plate is subject to periodic loads, in the other is fixed.

The final geometry of the problem is given in Fig. 1. The boundaries are denoted $\partial\Omega_i$, $i = 1, 2, 3, 4, 5, 6, 7$.

The boundary conditions play a fundamental role in dynamic models working iteratively. If the function that gives the displacements is denoted by u, in $\partial\Omega_l$ the Dirichlet condition is u(p, t) = p for all t ∈ [a, h], where [a, b] is the considered time interval (usually a = 0), in the part of the boundary where the specimen is fixed.

In the other parts of Ω the Dirichlet condition is not fixed, reflecting the fact that the iteration process begins with the original geometry but admits displacements in these boundaries given by the evolution of the deformation, evolution given also by the FE approximation.

Neumann conditions were imposed in the part of the boundary $\partial\Omega_j$ where the loads are working. Here some major considerations were made. To begin with, the stress model is based on linear elasticity, as mentioned. It is not a viscoelastic model [6] and thus not (yet) a fatigue model. The progress of the crack is simulated using the crack-growth velocity (CGV) [4] and does not arise as a consequence of involved physical laws.

Analog to the theory of geometric optic [7] it was assumed that the amplitude of the "waves" given by the function representing the periodic loads tends to cero. In geometric optic that assumption leads to geodesics as trajectories of photons, here it leads to constant loads along the process:

$$\frac{\partial u}{\partial n}(p,t) = K, \text{ in } \partial\Omega_j \qquad (1)$$

where n is the vector field of unitary vectors orthogonal to the boundary.

2.2. Combining stress analysis with empirical CGV

2.2.1. According to the premises of linear elasticity there should be no residual stresses at the beginning of the process [5, p. 200]. The Finite Element analysis is made under this premise, like many other´s FEM software. The authors are aware, however, that in the real situation this condition is not always fulfilled, giving rise to interesting experimental questions. Maybe the most interesting one is the question posed by the experimental fact, that we could also verify, that in some conditions, depending mainly on the residual stresses, the crack growth tends to a stationary situation [2], [8]. If the model here presented is generalized to elasto-plasticity, we are convinced that this case can be fruitfully analyzed.

2.2.2. The thermodynamics of the process has been abstracted. Also, no attempt was made to model the alterations in the area around the welding stripe where in the real case the temperature of the welding process could have had an influence [9].

2.2.3. The role of the body forces, mainly gravity, is assumed negligible compared to the other forces involved and it is thus also abstracted.

2.2.4. The CGV is an empirical standard estimate of the velocity of crack growth [1]. It is here used also to obtain an empirical estimate of the growth length,

Gl = CGV/time.

2.2.5. Under the considered premises, at every time step the stresses induced in the specimens depend on the forces applied and the geometry of the body. If, as mentioned, the forces at the boundary are considered constant, the changes in stresses for a growing crack manifest themselves as originated in the changing geometry of the specimen.

2.2.6. The last point, 2.2.5., is the main hypothesis of this work, together with the theoretical implications of the use of CGV. It implies that we know that the crack exists and grows at a certain (approximate) velocity that has been already empirically established, without needing to know which internal mechanisms generate the phenomenon of fatigue and fracture.

2.2.7. Under these hypothesis the problem can be so stated:

It is experimentally known that for concrete measured loads a crack begins and grows in the body represented by Ω (see Fig. 01). The growth of the crack is also experimentally established. Correlate this growth with the induced stresses and their evolution as the geometry of the body changes with the fracture?

2.3. A discrete dynamical procedure

The solution lays on the numerical procedure (FEM) itself. Given an elliptical problem defined on a body Ω:

$$P(u) = f, \qquad (2)$$

where P is an elliptic differential operator involving second order derivates and with Dirichlet and Neumann conditions, assume that a numerical discrete scheme for approximate solutions of (2) is obtained via MEF, giving as a result a polynomial numerical system represented by

$$\mathbf{M}\,\mathbf{u} = \mathbf{F} \quad (3)$$

Where **M** is a numerical matrix, i.e. a matrix with finite rational numbers as elements, calculated as numerical functions of the geometry of the discretized region Ω, of the interpolation and weight-functions and of the boundary values. **u** is the numerical vector of the unknown (approximate) values of the sought function u at the degrees of freedom. **F** is the corresponding right–side vector whoseelements are the scalar product of the elements of the original function f, f_i, with the interpolation functions φ_i.

The linear-elasticity dynamic problem corresponding to (2), considering that the body forces are abstracted, is

$$\mu\Delta u + (\lambda + \mu)\nabla div(u) = \rho_0 \frac{\partial^2 u}{\partial t^2} \quad (4)$$

where λ and μ are the Lamé constants and ρ_0 the density of Ω at the initial time [5, p. 201]. Because ρ_0 is different from 0, (4) can be expressed as

$$\frac{\partial^2 u}{\partial t^2} - P(u) = f \quad (5)$$

defined in Ω, with initial and boundary (Dirichlet and Neumann) conditions. In this case f is the constant function equal to cero, but the procedure is valid and presented in a more general form if f is different from cero.

Numerically a first approximation of u, u_0, is constructed for time t = 0, and then the ordinary second-order differential equation

$$\frac{\partial^2 u}{\partial t^2} - \mathbf{Mu} = \mathbf{F} \quad (6)$$

is solved via some finite-differences scheme. The term

$$\frac{\partial^2 u}{\partial t^2}$$

is usually discretized using a second-order finite difference scheme, because the sought values are the displacements at the nodes. But if this term is written as

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial t}\right) = \frac{\partial}{\partial t}\mathrm{v}$$

a first order scheme using the velocities at the nodes, $v_i = v(t_i)$, is obtained: Due to the fact that we dispose over an empirical estimation of the first time derivate of u (u'), the empirical crack-growth velocity, that we denote v_i, i = 0, 1, …,n, for each of the time steps, an empirical finite differences approximation of the acceleration is given by

$$u^{''} \approx \frac{\mathrm{v}_{i+1} - \mathrm{v}_i}{t_{i+1} - t_i} = \frac{\mathrm{v}_{i+1} - \mathrm{v}_i}{h} \quad (7)$$

considering a constant time interval *h*.

(7) is then used to update the left side of (6) at each time step in the following scheme:

1. Solve (3) with the initial boundary conditions and obtain a first approximation $\boldsymbol{u_0}$. Using the experimental data obtain a first estimate v_0 of the CGV. Put $\Omega = \Omega_0$.

2. Given the approximation u_i, calculate the i-th approximation of the acceleration

$$u^{''} \approx \frac{\mathrm{v}_{i+1} - \mathrm{v}_i}{t_{i+1} - t_i} = \frac{\mathrm{v}_{i+1} - \mathrm{v}_i}{h}$$

and solve the stress problem

$$P(u) = f + u_i^{''}$$

using FEM, defining the problem over the deformed region Ω_{i+1} obtained in the previous step and introducing manually the empirical (experimentally obtained) new length and width of the crack.

The sought approximation u_{i+1} for time step i+1 is then given by

$$\mathbf{Mu}_{i+1} = F + \frac{\mathrm{v}_{i+1} - \mathrm{v}_i}{h}, \text{ on } \Omega_{i+1}$$

With Dirichlet boundary conditions

$u(p, t) = u_0$, in $\partial\Omega_i$, i =1, 2… (fix points)

$u(p, t) = p_i$, $p_i = u_i(p)$ in $\partial\Omega_i$, i =1, 2…. p_i gives the new position (deformed configuration) of point p_{i-1}.

And with the same Neumann conditions:

$$\frac{\partial u}{\partial n}(p, t) = K, \text{ in } \partial\Omega_i$$

The series u_0, u_1, u_2, u_3, …, u_n gives then an approximation to the time evolution of the crack dynamically linked to the evolution of the stresses.

This kind of approximations, where the data play a fundamental role, can be called (numerical) *approximations of empirical dynamics*.

In this way all experimental information and the conceptual model are combined into a single procedure. A very complete simulation of the real process is obtained, allowing the researcher to conjugate the evolution of the stress with the growth and growth-rate of the crack.

2.4. Validation

The experimentally measured and the calculated deformed regions were compared. The metric employed is defined in Part 2. It measures the difference between a configuration Ω and another Ω'. The analysis showed adequate congruence between the experimental and the calculated configurations.

2.5. Introducing feedback from the experiments in the model

It was experimentally established that the asymmetry of the forces causes a deviation of the growth direction from the plane where the crack begins to grow. This can be simulated assuming that the deviation takes place along a second order curve, and for each point of the curve the CGV is the projection of the velocity of a particle on the curve over the original plane (where the crack would have remained if the forces were symmetric). The direction of the velocity along the curve is given by its derivate at the point. Given the projection and the direction, it is then possible to calculate the velocity of the crack growth along the curve, taking the asymmetry of the forces into consideration.

Simply because of the irregularity of the crack surface it is not clear which function could give a relatively good approximation of the real path of the fracture. Besides some maybe incidental coincidences with a brachistochrone curve, this kind of curve could be used because it is well known that certain displacements achieve a minimum of energy consumption along such a curve. But, as mentioned, thermodynamical considerations will be analyzed in future developments.

It is worth mentioning that a different approach can also be taken, assuming the fracture surface to be of a fractal

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nature. As have been shown [10], the fractal dimension of the surface can then be obtained experimentally, giving valuable information about the fracture processes involved.

We are certain that the systematic approach and development of the model here presented can help to reach insight into these and other questions.

III. Results and Conclusions

3.1. Results

Several sets of ten images simulating the crack growth were obtained, corresponding to a time step of one tenth of the time required for the crack to reach 20 mm of length. The first, last and one intermediate are shown for two of the four cases calculated.

The first, Set01, shows the crack growth and the corresponding stresses as the crack progresses linearly, without deviation. Set01 serves as a reference for the rest of the sets of images (See Figures 2 through 4).

Set02 shows the growth of the crack along a curve, assumed to be a numerical second order approximation to it, constructed with experimental data and simulating the deviation of the crack (See Figures 5 through 7).

The other two cases are presented in Part 2:

Set03 shows the rectilinear growth of the crack in the presence of an inhomogeneity simulating a stripe of welding.

Set04 shows the same situation but considering the deviation of the path of growth of the crack.

All settings are based in measured data corresponding to a series of experiments made by one of the authors (J. L. Ramírez et al.). See Part 2 [12].

3.2. Discussion and conclusions

Stress forces are the main cause of a fracture, it is certainly relevant to know how these forces act and distribute in the considered body along the process of crack growth. But until now no efficient scheme to combine the temporal and the spatial data was available. A solution is presented using numerical schemes.

Certainly, the scheme has still to be validated in other settings, but it is based in two robust tools, one empirical, CGV, and other, MEF, using efficiently the well-based knowledge of continuum mechanics. The dynamical combination is made here via a first order finite-differences scheme, an efficient numerical tool.

The procedure is robust enough and it can be used to explore and predict, mainly changing the values of the involved parameters in a consistent way.

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Author's Profile

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Professor for pure and applied Mechanics at the Faculty of Engineering, UNAM. Doctor in applied Mathematics. Doctor in Philosophy of Science.Hismain focus of research and teaching is themathematical and computational modeling ofphysical systems using differential equations and theFinite Element Method for discretization. Pure and applied (toinformatics) logic is his other point of interest. He has been working atthe UNAM 27 years. Besides he has worked in many fields of applicationof mathematics.

José Luis Ramírez Cruz

He is currently a Ph. D. candidate in Mechanical Engineering at UNAM in México City. His current research interest include materials fatigue, fracture mechanics and kinematics of mechanisms. He has teaching experience of 10 years.

Appendix

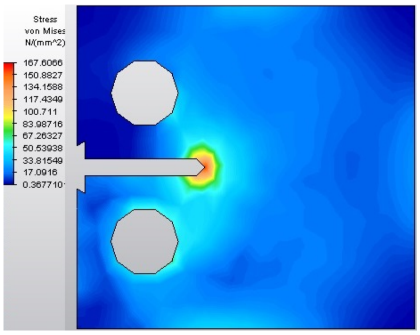

Fig.2. Set01 initial image.

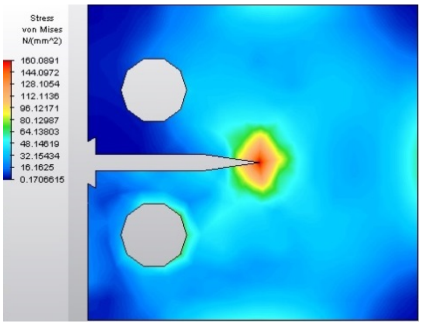

Fig.3. Set01 intermediate image.

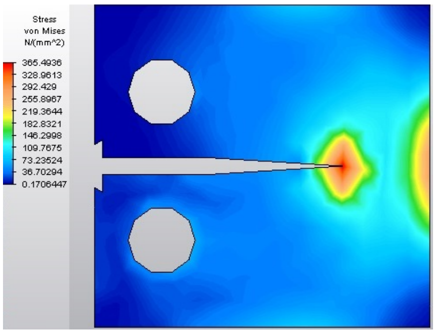

Fig.4. Set01 final image.

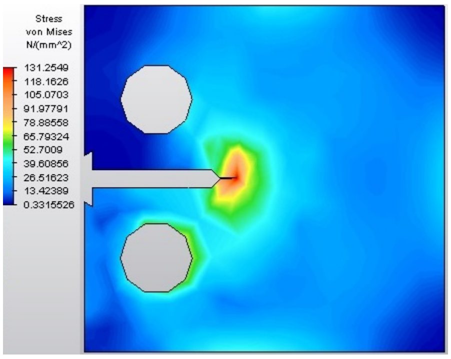

Fig.05. Set02 initial image.

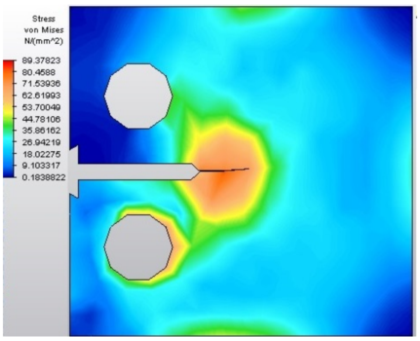

Fig.6. Set02 intermediate image.

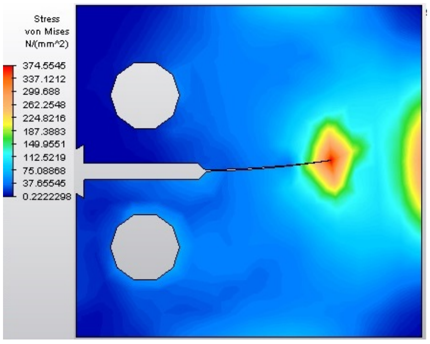

Fig.7. Set02 final image.