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Combine of B-Spline Galerkin Schemes with Change Weight Function

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***Abstract* - There exist many numerical methods forsolving differential equations. They differ in accuracy,performance and applicability. In this paper, we derive new numerical schemeswhich depend on type M of B-splines Galerkin method takes withweight function from type M-1 of B-splines, where M is integer number, for solving the modified equal width (MEW) wave equation,compared with analytic solution can be made and we investigate a linear stability analysiswhich is based on a Fourier (Von Neumann) method.**

***Keywords* – B-Spline Method, Galerkin Method, Von Neumann Method, Modified Equal Width Wave Equation.**

I. INTRODUCTION

Consider the (MEW) equation having normalized form [5]

$U_t + 3U^2U_x - \mu U_{xxt} = 0 \quad a \le x \le b,$ (1.1a)

subject to the following boundary conditions

$U(a,t) = 0, \quad U(b,t) = 0,$

$U_x(a,t) = 0, \quad U_x(b,t) = 0$

$U_{xx}(a,t) = U_{xx}(b,t) = 0, \; t > 0.$ (1.1b)

and the initial condition $U(x,0) = f(x)$, (1.1c)

the parameter μ is a positive constant and $f(x)$ islocalized disturbance inside interval $[a, b]$ with physical boundary conditions $U \to 0$ as $x \to \pm\infty$. The MEW equation (1.1a) has a solitary wave solution [8] of the form

$U(x,t) = Ʈ\, sech(\mathcal{K}[x - x_0 - \upsilon t]),$ (1.2a)

where the wave velocity $\upsilon = \frac{Ʈ^2}{2}$ and $\mathcal{K}^2 = 1/\mu$. This equation represents a single solitary wave of amplitude Ʈ, initially centered on x_0. The initial condition is taken as

$U(x,0) = Ʈ\, sech(\mathcal{K}[x - x_0]),$ (1.2b)

these solitary waves may have either a positive or a negative magnitude but all have positive velocities proportional to the square of their amplitude and, like the regularized long wave(RLW) equation, all have the same wave number $\mathcal{K} = \sqrt{(1/\mu)}$, thus all solitary waves have the same width. There is no forbidden range of positive velocities as occurs with the RLW equation. With these boundary conditions solutions of the MEW equation (1.1a) satisfy three invariant conditions given as:-

$$I_1 = \int_{-\infty}^{+\infty} U\,dx,$$

$$I_2 = \int_{-\infty}^{+\infty} (U^2 + \mu (U_x)^2)\,dx,$$

$$I_3 = \int_{-\infty}^{+\infty} U^4\,dx. \qquad (1.3)$$

The MEW equation based upon the equal width wave equation([2],[3]) which was suggested by[5] is used as a model partial differential equation for the simulation of one-dimensional wave propagation in nonlinear media with dispersion processes. This equation is related with the modified regularized long wave (MRLW) equation [1] and modified Korteweg-de Vries (MKdV) equation [4]. All the modified equations are nonlinear wave equations with cubic nonlinearities and all of them have solitary wave solutions, which are wave packets or pulses. These waves propagate in non-linear media by keeping wave forms and velocity even after interaction occurs.

II. APPROXIMATION OF THE MEW EQUATION BY B-SPLINE GALERKIN METHODS WITH DIFFERENT WEIGHT FUNCTION

2.1 Quadratic B-Spline Galerkin Method with Linear Weight Function

The quadratic B-spline $B_m(x)$ and its principle derivative which is definedby[7]vanishes outside the interval $[x_{m-1}, x_{m+2}]$,and takesthe weight function $W_1(x)$ linear B-spline.By using the local coordinate transformation [9]

$$h\eta = x - x_m, \qquad 0 \le \eta \le 1,$$

the linear B-spline shape functions for the typical element $[x_m, x_{m+1}]$ define by

$A_m = 1 - \eta, \qquad A_{m+1} = \eta,$

then, the weak form of (1.1a) is given by:

$$\int_a^b W_1 (U_t + 3U^2 U_x - \mu U_{xxt})\,dx = 0$$

so,

$$\int_0^1 W_1 (U_t + \frac{3}{h}\hat{U}^2 U_\eta - \frac{\mu}{h^2} U_{\eta\eta t})\,d\eta = 0$$

integrating by parts

$$\int_0^1 (W_1 U_t + \lambda W_1 U_\eta + \beta W_{1_\eta} U_{\eta t})\,d\eta = \beta W_1 U_{\eta t} \,|_0^1$$

(2.1)

substituting approximation

$$U_N(\eta,t)=\sum_{j=m-1}^{m+1} B_j(\eta)\gamma_j(t), \quad (2.2)$$

into (2.1), we obtain

$$\sum_{j=m-1}^{m+1}[(\int_0^1 A_i\, B_j+6A_i^{'}B_j^{'})d\eta-6A_iB_j^{'}\ |_0^1]\gamma_j^{\cdot e}$$

$$+\sum_{j=m-1}^{m+1}(\lambda\int_0^1 A_iB_j^{'}d\eta)\,\gamma_j^e=0,$$

which can be written in matrix form as follows:

$[X_{ij}^e+6(Y_{ij}^e-R_{ij}^e)]\gamma^{\cdot e}+\lambda Q_{ij}^e\gamma^e=0,$

where $\gamma^e=(\gamma_{m-1},\gamma_m,\gamma_{m+1})^T$ are the element parameters. The element matrices $X_{ij}^e, Y_{ij}^e, Q_{ij}^e$ and R_{ij}^e are rectangular $2\times 3$are given by the following integrals:

$$X_{ij}^e=\int_0^1 A_iB_j\,d\eta=\frac{1}{12}\begin{bmatrix}3 & 8 & 1\\ 1 & 8 & 3\end{bmatrix},$$

$$Y_{ij}^e=\int_0^1 A'_iB_j^{'}\,d\eta=\begin{bmatrix}1 & 0 & -1\\ -1 & 0 & 1\end{bmatrix},$$

$$Q_{ij}^e=\int_0^1 A_iB_j^{'}\,d\eta=\frac{1}{3}\begin{bmatrix}-2 & 1 & 1\\ -1 & -1 & 2\end{bmatrix}.$$

$$R_{ij}^e=A_iB_j^{'}\ |_0^1=\begin{bmatrix}2 & -2 & 0\\ 0 & -2 & 2\end{bmatrix},$$

where sufficesi takes only the values 1 and 2 and j takes values *m-1*,*m* and *m+1* for the typical element $[x_m,x_{m+1}]$.

A lumped value of λ can be defined as

$\lambda=\frac{3}{4h}(\gamma_{m-1}+2\gamma_m+\gamma_{m+1})^2,$

assembling all contributions from all element, we get the following matrix equation:

$[X_7+6(Y_7-R_7)]\gamma^{\cdot}+\lambda Q_7\gamma=0.$(2.3)

where $\gamma=(\gamma_{-1},\gamma_0,\gamma_1,\dots,\gamma_N)^T$ is a global element parameter. The matrices X_1,$Y_1$and $Q_1$are rectangular, penta-diagonal and row *m* of each has the following form:

$X_1=\frac{1}{12}(1,11,11,1,0),$

$Y_1=(-1,1,1,-1,0),$

$\lambda Q_1=(-\lambda_1,-\lambda_1-2\lambda_2,2\lambda_1+\lambda_2,\lambda_2,0),$

where,

$\lambda_1=\frac{3}{4h}(\gamma_{m-2}+2\gamma_{m-1}+\gamma_m)^2,$

$\lambda_2=\frac{3}{4h}(\gamma_{m-1}+2\gamma_m+\gamma_{m+1})^2,$

using the Crank-Nicholson approach $\gamma=\frac{1}{2}(\gamma^n+\gamma^{n+1})$ and the forward finite difference $\gamma^{\cdot}=\frac{\gamma^{n+1}-\gamma^n}{\Delta t}$ in (2.3) we obtain the following $(N+1)\times(N+2)$ matrix system

$$[X_1+6(Y_1-R_1)+\tfrac{\lambda\Delta t}{2}Q_1]\gamma^{n+1}=[X_1+6(Y_1-R_1)-\lambda\Delta t2Q1\gamma n, \quad (2.4)$$

to make the matrix equation be square we applying the boundary conditions (1.1b) to the system (2.3).

Remark 1: The initial vector of parameter $\gamma^0=(\gamma_0^0,\gamma_1^0,\dots,\gamma_N^0)$ must be determined to iterate system (2.4), the approximation

$$U_N(x,t)=\sum_{i=0}^{N} A_i(x)\omega_i(t), \quad (2.5)$$

is rewritten over the interval $[a,b]$ at time $t=0$ as follows:

$$U_N(x,0)=\sum_{m=0}^{N} A_m\,\omega_m^0,$$

$U(x,0)$ are required to satisfy the following relations at the mesh points x_m:

$U_N(x_m,0)=U(x_m,0)$, *m=0,1,…,N*

$U'_N(x_0,0)=U'(x_N,0)=0\,,$

$U''_N(x_0,0)=U''(x_N,0)=0,$

By this remark, the initial vector of parameter γ^0 is then determined asdetermined as

$$\begin{bmatrix}2 & -2 & & & \\ 1 & 1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & 1 & 1\\ & & & 2 & -2\end{bmatrix}\begin{bmatrix}\gamma_{-1}^0\\ \gamma_0^0\\ \vdots\\ \gamma_{N-1}^0\\ \gamma_N^0\end{bmatrix}=\begin{bmatrix}0\\ U(x_0)\\ \vdots\\ \vdots\\ U(x_{N-1})\\ U(x_N)\end{bmatrix}, \quad (2.6)$$

to solve this system, first reduce it to tri-diagonal matrix by eliminating the first equation from them and then apply Thomas algorithm[6].

2.2 Cubic B-Spline GalerkinMethod with Quadratic Weight Function

The cubic B-spline$C_m(x)$ and its two principle derivatives which are defined by[7]vanishes outside the interval $[x_{m-2},x_{m+2}]$,and take weight function $W_2(x)$ quadratic B-spline.By using the local coordinate transformation,the quadratic B-spline shape functions for the typical element $[x_m,x_{m+1}]$ defined by:-

$B_{m-1}=(1-\eta)^2,$

$B_m=1+2\eta-2\eta^2,$

$B_{m+1}=\eta^2,$

then,the weak form of (1.1a) is:

$$\int_0^1(W_2\,U_t+\lambda W_2U_\eta+6W_{2_\eta}U_{\eta t})d\eta=6W_2U_{\eta t}\ |_0^1 \quad 2.7)$$

substituting approximation

$$U_N(\eta,t)=\sum_{j_8=m-1}^{m+2} C_{j_8}(\eta)\sigma_{j_8}(t), \quad (2.8)$$

into integral equation (2.8), we get,

$$\sum_{j_8=m-1}^{m+2}[(\int_0^1 B_{i_8}\,C_{j_8}+6B_{i_8}^{'}C_{j_8}^{'})d\eta-6B_{i_8}C_{j_8}^{'}\ |_0^1]\sigma_{j_8}^{\cdot e}$$

$$+\sum_{j_8=m-1}^{m+2}(\lambda\int_0^1 B_{i_8}C_{j_8}^{'}d\eta)\,\sigma_{j_8}^e=0,$$

which can be written in matrix form as follows:

$[X^e_{i_8j_8} + \beta(Y^e_{i_8j_8} - R^e_{i_8j_8})]\sigma^{\cdot e} + \lambda Q^e_{i_8j_8}\sigma^e = 0.$

where $\sigma^e = (\sigma_{m-1}, \sigma_m, \sigma_{m+1}, \sigma_{m+2})^T$ are the element parameters. The element matrices $X^e_{i_8j_8}, Y^e_{i_8j_8}, Q^e_{i_8j_8}$ and $R^e_{i_8j_8}$ are rectangular 3×4 given by the following integrals:

$$X^e_{i_8j_8} = \int_0^1 B_{i_8}C_{j_8}\,d\eta = \frac{1}{60}\begin{bmatrix}10 & 71 & 38 & 1\\ 19 & 221 & 221 & 19\\ 1 & 38 & 71 & 10\end{bmatrix},$$

$$Y^e_{i_8j_8} = \int_0^1 B'_{i_8}C'_{j_8}\,d\eta = \frac{1}{2}\begin{bmatrix}3 & 5 & -7 & -1\\ -2 & 2 & 2 & -2\\ -1 & -7 & 5 & 3\end{bmatrix},$$

$$Q^e_{i_8j_8} = \int_0^1 B_{i_8}C'_{j_8}\,d\eta = \frac{1}{10}\begin{bmatrix}-6 & -7 & 12 & 1\\ -13 & -41 & 41 & 13\\ -1 & -12 & 7 & 6\end{bmatrix},$$

$$R^e_{i_8j_8} = B_{i_8}C'_{j_8}\,|_0^1 = 3\begin{bmatrix}1 & 0 & -1 & 0\\ 1 & -1 & -1 & 1\\ 0 & -1 & 0 & 1\end{bmatrix}$$

where suffices i_8 takes only the values 1,2,3 and j_8 takes values *m-1*,*m*,*m*+1 and *m*+2, for the typical element $[x_m, x_{m+1}]$. A lumped value for λ is defined by

$\lambda = \frac{3}{4h}(\sigma_{m-1} + 5\sigma_m + 5\sigma_{m+1} + \sigma_{m+2})^2,$

by assembling all contributions from all element, we get the following matrix equation:

$$[X_2 + \beta(Y_2 - R_2)]\sigma^{\cdot} + \lambda Q_2\sigma = 0. \qquad (2.9)$$

where $\sigma = (\sigma_{-1}, \sigma_0, \sigma_1, \dots, \sigma_N, \sigma_{N+1})^T$ is a global element parameter. The matrices X_2, Y_2 and Q_2 are rectangular, septa-diagonal and row of each has the following form:

$X_2 = \frac{1}{60}(1, 57, 302, 302, 57, 1, 0),$

$Y_2 = \frac{1}{2}(-1, -9, 10, 10, -9, -1, 0),$

$$\lambda Q_2 = \frac{1}{10}(-\lambda_1, -12\lambda_1 - 13\lambda_2, 7\lambda_1 - 41\lambda_2 - 6\lambda_3, 6\lambda_1 + 41\lambda_2 - 7\lambda_3, 13\lambda_2 + 12\lambda_3, \lambda_3, 0),$$

where,

$\lambda_1 = \frac{3}{4h}(\sigma_{m-2} + 5\sigma_{m-1} + 5\sigma_m + \sigma_{m+1})^2,$

$\lambda_2 = \frac{3}{4h}(\sigma_{m-1} + 5\sigma_m + 5\sigma_{m+1} + \sigma_{m+2})^2,$

$\lambda_3 = \frac{3}{4h}(\sigma_m + 5\sigma_{m+1} + 5\sigma_{m+2} + \sigma_{m+3})^2,$

using the Crank-Nicholson approach and the forward finite difference in (2.9)we obtain $(N+2)\times(N+3)$ matrix system

$$[X_2 + \beta(Y_2 - R_2) + \frac{\lambda\Delta t}{2}Q_2]\sigma^{n+1} = [X_2 + \beta(Y_2 - R_2) - \lambda\Delta t2\, Q2\sigma n. \qquad (2.10)$$

Applying the boundary conditions to system (2.9) we make the matrix equation square.By Remark 1, the initial vector of parameter σ^0 is then determined as

$$\begin{bmatrix}3 & 0 & -3 & & & \\ 1 & 4 & 1 & & & \\ & & \ddots & & & \\ & & \ddots & \ddots & & \\ & & & 1 & 4 & 1\\ & & & 3 & 0 & -3\end{bmatrix}\begin{bmatrix}\sigma^0_{-1}\\ \sigma^0_0\\ \vdots\\ \sigma^0_N\\ \sigma^0_{N+1}\end{bmatrix} = \begin{bmatrix}0\\ U(x_0)\\ \vdots\\ \vdots\\ U(x_N)\\ 0\end{bmatrix}, \qquad (2.11)$$

to solve this matrix equation, first reduce it to tri-diagonal form by eliminating the first and last equations and then apply the Thomas algorithm.

2.3 Quartic B-Spline GalerkinMethod with Cubic Weight Function

The quartic B-spline $D_m(x)$ and its two principle derivatives which are defined in [7] vanishes outside the interval $[x_{m-2}, x_{m+3}]$,and take weight function $W_3(x)$ cubic B-spline.By using the local coordinate transformation, we give the cubic B-spline shape functions for the typical element $[x_m, x_{m+1}]$ in

$C_{m-1} = (1-\eta)^3,$

$C_m = 1 + 3(1-\eta) + 3(1-\eta)^2 - 3(1-\eta)^3,$

$C_{m+1} = 1 + 3\eta + 3\eta^2 - 3\eta^3,$

$C_{m+2} = \eta^3$.

then, the weak form of (1.1a) is:

$$\int_0^1 (W_3 U_t + \lambda W_3 U_\eta + \beta W_{3_\eta} U_{\eta t})d\eta = \beta W_3 U_{\eta t}\,|_0^1, \qquad (2.12)$$

substituting approximation

$$U_N(\eta, t) = \sum_{j_9=m-2}^{m+2} D_{j_9}(\eta)\rho_{j_9}(t), \qquad (2.13)$$

into integral equation (2.12), we get,

$$\sum_{j_9=m-2}^{m+2}[(\int_0^1 C_{i_9}D_{j_9} + \beta C'_{i_9}D'_{j_9})d\eta - \beta C_{i_9}D'_{j_9}\,|_0^1]\rho^{\cdot e}_{j_9} + \sum_{j_9=m-2}^{m+2}(\lambda\int_0^1 C_{i_9}D'_{j_9}d\eta)\,\rho^e_{j_9} = 0,$$

which can be written in matrix form as follows:

$[X^e_{i_9j_9} + \beta(Y^e_{i_9j_9} - R^e_{i_9j_9})]\rho^{\cdot e} + \lambda Q^e_{i_9j_9}\rho^e = 0,$

where $\rho^e = (\rho_{m-2}, \rho_{m-1}, \rho_m, \rho_{m+1}, \rho_{m+2})^T$ are the element parameters. The element matrices $X^e_{i_9j_9}, Y^e_{i_9j_9}, R^e_{i_9j_9}$ and $Q^e_{i_9j_9}$ are rectangular 4×5 given by the following integrals:

$$X^e_{i_9j_9} = \int_0^1 C_{i_9}D_{j_9}\,d\eta = \frac{1}{280}\begin{bmatrix}35 & 594 & 892 & 158 & 1\\ 211 & 4794 & 10196 & 3190 & 89\\ 89 & 3190 & 10196 & 4794 & 211\\ 1 & 158 & 892 & 594 & 35\end{bmatrix},$$

$$Y^e_{i_9j_9} = \int_0^1 C'_{i_9}D'_{j_9}\,d\eta$$

$$=\frac{1}{5}\begin{bmatrix}10 & 61 & -33 & -37 & -1\\ 9 & 141 & 33 & -165 & -18\\ -18 & -165 & 33 & 141 & 9\\ -1 & -37 & -33 & 61 & 10\end{bmatrix},$$

$$R^e_{i_9j_9}=(C_{i_9}D'_{j_9})\,|_0^1=\begin{bmatrix}4 & 12 & -12 & -4 & 0\\ 16 & 44 & -60 & -4 & 4\\ 4 & -4 & -60 & 44 & 16\\ 0 & -4 & -12 & 12 & 4\end{bmatrix},$$

$$Q^e_{i_9j_9}=\int_0^1 C_{i_9}D'_{j_9}\,d\eta$$

$$=\frac{1}{35}\begin{bmatrix}-20 & -109 & 69 & 59 & 1\\ -129 & -1059 & 255 & 873 & 60\\ -60 & -873 & -255 & 1059 & 129\\ -1 & -59 & -69 & 109 & 20\end{bmatrix},$$

where suffices $i_9$takes only the values 1,2,3,4 and $j_9$takes values m-2, m-1,m, m+1 and m+2 for the typical element $[x_m, x_{m+1}]$. A lumped value defined as

$\lambda=\frac{3}{4h}(\rho_{m-2}+12\rho_{m-1}+22\rho_m+12\rho_{m+1}+\rho_{m+2})^2.$

By assembling all contributions from all elementwe get the following matrix equation:

$$[X_3+\beta(Y_3-R_3)]\rho^{\cdot}+\lambda\,Q_3\rho=0. \qquad (2.14)$$

where $\rho=(\rho_{-2},\rho_{-1},\dots,\rho_N,\rho_{N+1})^T$ is a global element parameter. The matrices X_3,Y_3 and $\lambda\,Q_3$ are rectangular, nonic-diagonal and row of each has the following form:

$X_3=\frac{1}{280}(1,247,4293,15619,15619,4293,247,1,0),$

$Y_3=\frac{1}{5}(-1,-55,-189,245,245,-189,-55,-1,0\,),$

$$\lambda\,Q_3=\frac{1}{35}(-\lambda_1,-59\lambda_1-60\lambda_2,-69\lambda_1-873\lambda_2-129\lambda_3,109\lambda_1-255\lambda_2-1059\lambda_3-20\lambda_4,20\lambda_1+1059\lambda_2+255\lambda_3-109\lambda_4,129\lambda_2+873\lambda_3+69\lambda_4,\quad 60\lambda_3+59\lambda_4,\lambda_4,0)$$

where,

$\lambda_1=\frac{3}{4h}(\rho_{m-3}+12\rho_{m-2}+22\rho_{m-1}+12\rho_m+\rho_{m+1})^2,$

$\lambda_2=\frac{3}{4h}(\rho_{m-2}+12\rho_{m-1}+22\rho_m+12\rho_{m+1}+\rho_{m+2})^2,$

$\lambda_3=\frac{3}{4h}(\rho_{m-1}+12\rho_m+22\rho_{m+1}+12\rho_{m+2}+\rho_{m+3})^2,$

$\lambda_4=\frac{3}{4h}(\rho_m+12\rho_{m+1}+22\rho_{m+2}+12\rho_{m+3}+\rho_{m+4})^2,$

using the Crank-Nicholson approachfor $\rho$and for $\rho^{\cdot}$ the forward finite difference in (2.14) we obtain $(N$+3)× $(N+4)$ matrix system

$$[X_3+\beta(Y_3-R_3)+\tfrac{\lambda\Delta t}{2}Q_3]\rho^{n+1}=[X_3+\beta(Y_3-R_3)-\lambda\Delta t2\,Q3\rho n, \qquad (2.15)$$

by applying the boundary conditions to (2.14) we make the matrix equation square. By Remark 1, the initial vector of parameter ρ^0 is then determined as

$$\begin{bmatrix}12 & -12 & -12 & 12 & & & & \\ 4 & 12 & -12 & 4 & & & & \\ 1 & 11 & 11 & 1 & & & & \\ & & \ddots & \ddots & & & & \\ & & & \ddots & \ddots & & & \\ & & & 1 & 11 & 11 & 1 \\ & & & 4 & 12 & -12 & 4 \\ & & & 12 & -12 & -12 & 12\end{bmatrix}\begin{bmatrix}\rho^0_{-2}\\ \rho^0_{-1}\\ \rho^0_0\\ \vdots\\ \vdots\\ \rho^0_N\\ \rho^0_{N+1}\end{bmatrix}=\begin{bmatrix}0\\ 0\\ U(x_0\\ \vdots\\ \vdots\\ U(x_N)\\ 0\end{bmatrix}, \qquad (2.16)$$

to solve this system, first reduce it to four-diagonal form by eliminating the first pair and last equations and then apply Thomas algorithm.

2.4 Quintic B-Spline GalerkinMethod with Quartic Weight Function

The quintic B-spline$E_m(x)$ and its two principle derivatives which are defined in [7] vanishes outside the interval $[x_{m-3}, x_{m+3}]$,and take weight function $W_4(x)$ quartic B-spline.By using the local coordinate transformation, we give the quartic B-spline shape functions for the typical element $[x_m, x_{m+1}]$ in

$\mathrm{D}_{m-2}=(1-\eta)^4,$

$\mathrm{D}_{m-1}=(2-\eta)^4-5(1-\eta)^4,$

$\mathrm{D}_{m}\;\;=(3-\eta)^4-5(2-\eta)^4+10(1-\eta)^4,$

$\mathrm{D}_{m+1}=(1+\eta)^4-5\eta^4,$

$\mathrm{D}_{m+2}=\eta^4,$

then, the weak form of (1.1a) is:

$$\int_0^1[W_4\,U_t+\lambda W_4U_\eta+\beta W_{4_\eta}U_{\eta t})d\eta=\beta W_4U_{\eta t}\;|_0^1 \qquad (2.17)$$

substituting approximation

$$U_N(\eta,t)=\sum_{j_{10}=m-2}^{m+3}E_{j_{10}}(\eta)\vartheta_{j_{10}}(t), \qquad (2.18)$$

into integral equation (2.17), we get,

$$\sum_{j_{10}=m-2}^{m+3}[(\int_0^1 D_{i_{10}}E_{j_{10}}+\beta D'_{i_{10}}E'_{j_{10}})d\eta-\beta D_{i_{10}}E'_{j_{10}}\;|_0^1]\vartheta^{\cdot e}_{j_{10}}$$
$$+\sum_{j_{10}=m-2}^{m+3}(\lambda\int_0^1 D_{i_{10}}E'_{j_{10}}d\eta)\,\vartheta^e_{j_{10}}=0,$$

which can be written in matrix form as follows:

$[X^e_{i_{10}j_{10}}+\beta(Y^e_{i_{10}j_{10}}-R^e_{i_{10}j_{10}})]\vartheta^{\cdot e}+\lambda Q^e_{i_{10}j_{10}}\vartheta^e=0.$

where $\vartheta^e=(\vartheta_{m-2},\vartheta_{m-1},\vartheta_m,\vartheta_{m+1,}\vartheta_{m+2},\vartheta_{m+3})^T$ are the element parameters. The element matrices $X^e_{i_{10}j_{10}}, Y^e_{i_{10}j_{10}}, R^e_{i_{10}j_{10}}$ and $Q^e_{i_{10}j_{10}}$ are rectangular (5×6) given by the following integrals:

$$X^e_{i_{10}j_{10}}=\int_0^1 D_{i_{10}}E_{j_{10}}\,d\eta$$

$$=\frac{1}{1260}\begin{bmatrix}126 & 4747 & 15962 & 8772 & 632 & 1\\ 1931 & 89797 & 376002 & 281662 & 36467 & 381\\ 2601 & 155637 & 839682 & 839682 & 155637 & 2601\\ 381 & 36467 & 281662 & 376002 & 89797 & 1931\\ 1 & 632 & 8772 & 15962 & 4747 & 126\end{bmatrix},$$

$$Y^e_{i_{10}j_{10}} = \int_0^1 D'_{i_{10}} E'_{j_{10}}\, d\eta$$

$$= \frac{1}{14}\begin{bmatrix} 35 & 559 & 298 & -734 & -157 & -1 \\ 176 & 4024 & 5104 & -6272 & -2944 & -88 \\ -122 & -1482 & 1604 & 1604 & -1482 & -122 \\ -88 & -2944 & -6272 & 5104 & 4024 & 176 \\ -1 & -157 & 734 & 298 & 559 & 35 \end{bmatrix}$$

$$R^e_{i_{10}j_{10}} = \left(D_{i_{10}} E'_{j_{10}}\right)\Big|_0^1$$

$$= \begin{bmatrix} 5 & 50 & 0 & -50 & -5 & 0 \\ 55 & 545 & -50 & -550 & -5 & 5 \\ 55 & 495 & -550 & -550 & 495 & 55 \\ 5 & -5 & -550 & -50 & 545 & 55 \\ 0 & -5 & -50 & 0 & 50 & 5 \end{bmatrix}$$

$$Q^e_{i_{10}j_{10}} = \int_0^1 D_{i_{10}} E'_{j_{10}}\, d\eta$$

$$= \frac{1}{126}\begin{bmatrix} -70 & -1051 & -460 & 1330 & 250 & 1 \\ -1121 & -21689 & -20186 & 31550 & 11195 & 251 \\ -1581 & -41415 & -67434 & 67434 & 41415 & 1581 \\ -251 & -11195 & -31550 & 20186 & 21689 & 1121 \\ -1 & -250 & -1330 & 460 & 1051 & 70 \end{bmatrix},$$

where suffices i_{10} takes only the values 1,2,3,4,5 and j_{10} takes values m-2, m-1,m, m+1, m+2 and m+3 for the typical element $[x_m, x_{m+1}]$. A lumped value is defined as

$$\lambda = \frac{3}{4h}\left(\vartheta_{m-2} + 26\vartheta_{m-1} + 66\vartheta_m + 26\vartheta_{m+1} + \vartheta_{m+2}\right)^2,$$

by assembling all contributions from all elementwe get the following matrix equation:

$$[X_4 + \beta(Y_4 - R_4)]\vartheta^{\cdot} + \lambda\, Q_4 \vartheta = 0, \qquad (2.19)$$

where $\vartheta = (\vartheta_{-2}, \vartheta_{-1}, \dots, \vartheta_N, \vartheta_{N+1}, \vartheta_{N+2})^T$ is a global element parameter. The matrices X_4,$Y_4$and $\lambda\, Q_4$ are rectangular, 11-diagonal and row of each has the following form:

$$X_4 = \frac{1}{1260}(1, 1013, 47840, 455192, 13103540,, 13103540, 455192, 47840, 1013, 1, 0\,)$$

$$Y_4 = \frac{1}{14}(-1, -245, -3800, -7280, 11326, 11326, -7280,\ -3800,\ -245, -1, 0),$$

$$\begin{aligned}\lambda\, Q_4 = \frac{1}{126}(&-\lambda_1, -250\lambda_1 - 251\lambda_2, -1330\lambda_1 \\ &-11195\lambda_2 - 1581\lambda_3, 406\lambda_1 - 31550\lambda_2 - 41415\lambda_3 \\ &-1121\lambda_4, 1051\lambda_1 + 20186\lambda_2 - 67434\lambda_3 - 21689\lambda_4 \\ &-70\lambda_5, 70\lambda_1 + 21689\lambda_2 + 67434\lambda_3 - 20186\lambda_4 \\ &-1051\lambda_5, 1121\lambda_2 + 41415\lambda_3 + 31550\lambda_4 \\ &+1330\lambda_5, 1581\lambda_3 + 11195\lambda_4 + 1330\lambda_5, 251\lambda_4 \\ &+250\lambda_5,\ \lambda_5, 0)\end{aligned}$$

where,

$\lambda_1 = \frac{3}{4h}(\vartheta_{m-3} + 27\vartheta_{m-2} + 92\vartheta_{m-1} + 92\vartheta_m + 27\vartheta_{m+1} + \vartheta_{m+2})^2,$

$\lambda_2 = \frac{3}{4h}(\vartheta_{m-2} + 27\vartheta_{m-1} + 92\vartheta_m + 92\vartheta_{m+1} + 27\vartheta_{m+2} + \vartheta_{m+3})^2,$

$\lambda_3 = \frac{3}{4h}(\vartheta_{m-1} + 27\vartheta_m + 92\vartheta_{m+1} + 92\vartheta_{m+2} + 27\vartheta_{m+3} + \vartheta_{m+4})^2,$

$\lambda_4 = \frac{3}{4h}(\vartheta_m + 27\vartheta_{m+1} + 92\vartheta_{m+2} + 92\vartheta_{m+3} + 27\vartheta_{m+4} + \vartheta_{m+5})^2,$

$\lambda_5 = \frac{3}{4h}(\vartheta_{m+1} + 27\vartheta_{m+2} + 92\vartheta_{m+3} + 92\vartheta_{m+4} + 27\vartheta_{m+5} + \vartheta_{m+6})^2,$

using the Crank-Nicholson approach and the forward finite difference in (2.19) we obtain $(N$+4)× $(N+5)$ matrix system

$$\left[X_4 + \beta(Y_4 - R_4) + \tfrac{\lambda\Delta t}{2} Q_4\right]\vartheta^{n+1} = \left[X_4 + \beta(Y_4 - R_4) - \tfrac{\lambda\Delta t}{2} Q_4\right]\vartheta^n, \qquad (2.20)$$

by applying the boundary conditions to (2.20) we make the matrix equation square. By Remark 1, the initial vector of parameter ϑ^0 is then determined as

$$\begin{bmatrix} 20 & 40 & -120 & 40 & 20 & & & & \\ 5 & 50 & 0 & -50 & -5 & & & & \\ 1 & 26 & 66 & 26 & 1 & & & & \\ & & & \ddots & \ddots & & & & \\ & & & \ddots & \ddots & & & & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & 5 & 50 & 0 & -50 & -5 \\ & & & 20 & 40 & -120 & 40 & 20 \end{bmatrix} \begin{bmatrix} \vartheta^0_{-2} \\ \vartheta^0_{-1} \\ \vartheta^0_0 \\ \vdots \\ \vdots \\ \vartheta^0_N \\ \vartheta^0_{N+1} \\ \vartheta^0_{N+2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ U(x_0 \\ \vdots \\ \vdots \\ U(x_N) \\ 0 \\ 0 \end{bmatrix}, \qquad (2.21)$$

to solve this system, first reduce it to penta-diagonal form by eliminating the first and last pair of equations and then apply Thomas algorithm.

2.5 Sextic B-Spline Galerkin Method with Quintic Weight Function

The sextic B-spline$F_m(x)$ and its two principle derivatives which are defined in [7] vanishes outside the interval $[x_{m-3}, x_{m+4}]$,and take weight function $W_5(x)$ quintic B-spline. By using the local coordinate transformation, we give the quintic B-spline shape functions for the typical element $[x_m, x_{m+1}]$ in

$$\begin{aligned}
E_{m-2} &= (1-\eta)^5, \\
E_{m-1} &= (2-\eta)^5 - 6(1-\eta)^5, \\
E_m &= (3-\eta)^5 - 6(2-\eta)^5 + 15(1-\eta)^5, \\
E_{m+1} &= (4-\eta)^5 - 6(3-\eta)^5 + 15(2-\eta)^5 - 20(1-\eta)^5, \\
E_{m+2} &= (5-\eta)^5 - 6(4-\eta)^5 + 15(3-\eta)^5 - 20(2-\eta)^5 + 15(1-\eta)^5, \\
E_{m+2} &= \eta^5,
\end{aligned}$$

then the weak form of (1.1a) is:

$$\int_0^1 [W_5\, U_t + \lambda W_5 U_\eta + \beta W_{5_\eta} U_{\eta t})d\eta = \beta W_5 U_{\eta t}\ \Big|_0^1 \qquad (2.22)$$

Substituting approximation

$$U_N(\eta, t) = \sum_{j_{11}=m-3}^{m+3} F_{j_{11}}(\eta)\tau_{j_{11}}(t), \qquad (2.23)$$

into integral equation (2.22), we get,

$$\sum_{j_{11}=m-3}^{m+3}[(\int_0^1 E_{i_{11}}F_{j_{11}}+\beta E'_{i_{11}}F'_{j_{11}})d\eta-\beta E_{i_{11}}F'_{j_{j_{11}}}|_0^1]\tau^{\cdot e}_{j_{11}}$$
$$+\sum_{j_9=m-3}^{m+3}(\lambda\int_0^1 E_{i_{11}}F'_{j_{11}}d\eta)\,\tau^e_{j_{11}}=0,$$

which can be written in matrix form as follows:

$$[X^e_{i_{11}j_{11}}+\beta(Y^e_{i_{11}j_{11}}-R^e_{i_{11}j_{11}})]\tau^{\cdot e}+\lambda Q^e_{i_{11}j_{11}}\tau^e=0.$$

where $\tau^e=(\tau_{m-3},\tau_{m-2},\tau_{m-1},\tau_m,\tau_{m+1},\tau_{m+2},\tau_{m+3})^T$ are the element parameters. The element matrices $X^e_{i_{11}j_{11}}, Y^e_{i_{11}j_{11}}, R^e_{i_{11}j_{11}}$ and $Q^e_{i_{11}j_{11}}$ are rectangular (6×7) given by the following integrals:

$$X^e_{i_{11}j_{11}}=\int_0^1 E_{j_{11}}F_{j_{11}}\,d\eta=\frac{1}{5544}$$
$$\begin{bmatrix} 462 & 36959 & 244205 & 304250 & 76900 & 2503 & 1\\ 16171 & 1537535 & 11886590 & 17975130 & 6128395 & 375559 & 1580\\ 51014 & 5748218 & 52521800 & 96528940 & 42334750 & 3704026 & 25812\\ 25812 & 3704026 & 42334750 & 96528940 & 52521800 & 5748218 & 51014\\ 1580 & 375559 & 6128395 & 17975130 & 11886590 & 1537535 & 16171\\ 1 & 2503 & 76900 & 304250 & 244205 & 36959 & 462 \end{bmatrix}$$

$$Y^e_{i_{11}j_{11}}=\int_0^1 E'_{i_{11}}F'_{j_{11}}\,d\eta$$
$$=\frac{1}{42}\begin{bmatrix} 126 & 4621 & 11215 & -7190 & -8140 & -631 & -1\\ 1805 & 83245 & 274990 & -87150 & -237055 & -35455 & -380\\ 670 & 65170 & 397840 & 94340 & -438850 & -116950 & -2220\\ -2220 & -116950 & -438850 & 94340 & 397840 & 65170 & 670\\ -380 & -35455 & -237055 & -87150 & 274990 & 83245 & 1805\\ -1 & -631 & -8140 & -7190 & 11215 & 4621 & 126 \end{bmatrix}$$

$$R^e_{i_{11}j_{11}}=\left(E_{i_{11}}F'_{j_{j_{11}}}\right)|_0^1=$$
$$\begin{bmatrix} 6 & 150 & 240 & -240 & -150 & -6 & 0\\ 156 & 3894 & 6090 & -6480 & -3660 & -6 & 6\\ 396 & 9744 & 11940 & -22080 & -3660 & 3504 & 156\\ 156 & 3504 & -3660 & -22080 & 11940 & 9744 & 396\\ 6 & -6 & -3660 & -6480 & 6090 & 3894 & 156\\ 0 & -6 & -150 & -240 & 240 & 150 & 6 \end{bmatrix}$$

$$Q^e_{i_{11}j_{11}}=\int_0^1 E_{i_{11}}F'_{j_{j_{11}}}\,d\eta=\frac{1}{462}$$
$$\begin{bmatrix} -252 & -8861 & -20445 & 14060 & 14480 & 1017 & 1\\ -9113 & -388303 & -1161290 & 486520 & 950545 & 120623 & 1018\\ -29558 & -1529148 & -5905750 & 861980 & 5530290 & 1056688 & 15498\\ -15498 & -1056688 & -5530290 & -861980 & 5905750 & 1529148 & 29558\\ -1018 & -120623 & -950545 & -486520 & 1161290 & 388303 & 9113\\ -1 & -1017 & -14480 & -14060 & 20445 & 8861 & 252 \end{bmatrix}$$

where suffices i_{11} takes only the values 1,2,3,4,5,6 and j_{10} takes values m-3, m-2, m-1,m, m+1 and m+2 for the typical element $[x_m, x_{m+1}]$. A lumped value is defined as $\lambda=\frac{3}{4h}(\tau_{m-3}+57\tau_{m-2}+302\tau_{m-1}+302\tau_m+57\tau_{m+1}+\tau_{m+2})^2$.

By assembling all contributions from all element we get the following matrix equation:

$[X_5+\beta(Y_5-R_5)]\tau^{\cdot}+\lambda Q_5\tau=0,$ (2.24)

where $\tau=(\tau_{-3},\tau_{-2},\tau_{-1},\ldots,\tau_N,\tau_{N+1},\tau_{N+2})^T$ is a global element parameter. The matrices X_5,Y_5 and λQ_5 are rectangular, nonic-diagonal and row of each has the following form:

$$X_5=\frac{1}{5544}(1,4083,478271,10187685,66318474,162512286,162512286,66318474,10187685,478271,4083,1,0)$$

$$Y_5=\frac{1}{42}(-1,-1011,-45815,-360525,-447810,855162,855162,-447810,-360525,-45815,-1011,-1,0)$$

$$\lambda Q_5=\frac{1}{462}(-\lambda_1,-1017\lambda_1-1018\lambda_2,-14480\lambda_1-120623\lambda_2-15498\lambda_3,-14060\lambda_1-950545\lambda_2-1056688\lambda_3-29558\lambda_4,20445\lambda_1-486520\lambda_2-5530290\lambda_3-1529148\lambda_4-9113\lambda_5,8861\lambda_1+1161290\lambda_2-861980\lambda_3-5905750\lambda_4-388303\lambda_5-252\lambda_6,252\lambda_1+388303\lambda_2+5905750\lambda_3+861980\lambda_4-1161290\lambda_5-8861\lambda_6,9113\lambda_2+1529148\lambda_3+5530290\lambda_4+486520\lambda_5-20445\lambda_6,29558\lambda_3+1056688\lambda_4+950545\lambda_5+14060\lambda_6,15498\lambda_4+120623\lambda_5+14480\lambda_6,1018\lambda_5+1017\lambda_6,\lambda_6,0)$$

where,

$\lambda_1=\frac{3}{4h}(\tau_{m-3}+58\tau_{m-2}+359\tau_{m-1}+604\tau_m+$ *359τm+1+58τm+2+τm+3)2*,

$\lambda_2=\frac{3}{4h}(\tau_{m-2}+58\tau_{m-1}+359\tau_m+604\tau_{m+1}+$ *359τm+2+58τm+3+τm+4)2*,

$\lambda_3=\frac{3}{4h}(\tau_{m-1}+58\tau_m+359\tau_{m+1}+604\tau_{m+2}+$ *359τm+3+58τm+4+τm+5)2*,

$\lambda_4=\frac{3}{4h}(\tau_m+58\tau_{m+1}+359\tau_{m+2}+604\tau_{m+3}+$ *359τm+4+58τm+5+τm+6)2*,

$\lambda_5=\frac{3}{4h}(\tau_{m+1}+58\tau_{m+2}+359\tau_{m+3}+604\tau_{m+4}+$ *359τm+5+58τm+6+τm+7)2*,

$\lambda_6=\frac{3}{4h}(\tau_{m+2}+58\tau_{m+3}+359\tau_{m+4}+604\tau_{m+5}+$ *359τm+6+58τm+7+τm+8)2*,

using the Crank-Nicholson approach for τ and the forward finite difference for $\tau^{\cdot}$ in (2.24) obtain $(N+5)\times(N+6)$ matrix system

$[X_5+\beta(Y_5-Z_5)+\frac{\lambda\Delta t}{2}Q_5]\tau^{n+1}=[X_5+\beta(Y_5-Z_5)-$ *λΔt2Q5τn*, (2.25) by applying the boundary conditions to (2.24) we make the matrix equation square. By Remark 1, the initial vector of parameterτ^0 is then determined as

$$\begin{bmatrix} 30 & 270 & -300 & -300 & 270 & 30 & & & & \\ 6 & 150 & 240 & -240 & -150 & -6 & & & & \\ 1 & 57 & 302 & 302 & 57 & 1 & & & & \\ & & \ddots & \ddots & \ddots & & & & & \\ & & & & 1 & 57 & 302 & 302 & 57 & 1 \\ & & & & 6 & 150 & 240 & -240 & -150 & -6 \\ & & & & 30 & 270 & -300 & -300 & 270 & 30 \end{bmatrix} \begin{bmatrix} \tau_{-3}^0 \\ \tau_{-2}^0 \\ \tau_{-1}^0 \\ \tau_0^0 \\ \vdots \\ \vdots \\ \tau_N^0 \\ \tau_{N+1}^0 \\ \tau_{N+2}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ U(x_0) \\ \vdots \\ \vdots \\ \vdots \\ U(x_N) \\ 0 \\ 0 \end{bmatrix} \qquad (2.26)$$

to solve this system, first reduce it to six-diagonal form by eliminating the first three and last pair of equations and then apply Thomas algorithm.
The numerical results of previous kinds of Galerkin B-spline with different weight function are shown in Table (1) and Figure (1).

III. Stability Analysis of Galerkin B-Spline Methods with Different Weight Function

3.1 Stability of Quadratic B-Spline GalerkinMethod with Linear B-Spline as a Weight Function

A typical member of the matrix system (2.4) can be written in terms of the nodal parameters γ_m^n as

$$v_1\gamma_{m-2}^{n+1} + v_2\gamma_{m-1}^{n+1} + v_3\gamma_m^{n+1} + v_4\gamma_{m+1}^{n+1} = v_4\gamma_{m-2}^n + v_3\gamma_{m-1}^n + v_2\gamma_m^n + v_1\gamma_{m+1}^n,$$

where

$$v_1 = \tfrac{1}{12} - \beta - \tfrac{\lambda\Delta t}{6}, \qquad v_2 = \tfrac{11}{12} + \beta - \tfrac{3\lambda\Delta t}{6},$$
$$v_3 = \tfrac{11}{12} + \beta + \tfrac{3\lambda\Delta t}{6}, \qquad v_4 = \tfrac{1}{12} - \beta + \tfrac{\lambda\Delta t}{6}.$$

Substitution of $\gamma_m^n = \ddot{Y}_{18}^n e^{i\beta m h}$, leads to

$$\ddot{Y}_{18}\{v_1 e^{-2i\beta h} + v_2 e^{-i\beta h} + v_3 + v_4 e^{i\beta h}\} = v_4 e^{-2i\beta h} + v_3 e^{-i\beta h} + v_2 + v_1 e^{i\beta h},$$

simplifying the above equation, we get

$$\ddot{Y}_{18}^n = \frac{A_{10} - iB_{10}}{C_{10} + iD_{10}},$$

where ,

$$A_{10} = \left(\tfrac{11}{12} + \beta - \tfrac{3\lambda\Delta t}{6}\right) + \left(1 + \tfrac{2\lambda\Delta t}{6}\right)\cos(h\beta) + \left(\tfrac{1}{12} - \beta + \tfrac{\lambda\Delta t}{6}\right)\cos(2h\beta),$$

$$B_{10} = \left(-\tfrac{10}{12} - 2\beta - \tfrac{4\lambda\Delta t}{6}\right)\sin(h\beta) + \left(-\tfrac{1}{12} + \beta - \tfrac{\lambda\Delta t}{6}\right)\sin(2h\beta),$$

$$C_{10} = \left(\tfrac{11}{12} + \beta + \tfrac{3\lambda\Delta t}{6}\right) + \left(1 - \tfrac{2\lambda\Delta t}{6}\right)\cos(h\beta) + \left(\tfrac{1}{12} - \beta - \tfrac{\lambda\Delta t}{6}\right)\cos(2h\beta),$$

$$D_{10} = \left(-\tfrac{10}{12} - 2\beta + \tfrac{4\lambda\Delta t}{6}\right)\sin(h\beta) + \left(-\tfrac{1}{12} + \beta + \tfrac{\lambda\Delta t}{6}\right)\cos(2h\beta),$$

after simplification, we obtain that $|\ddot{Y}_{18}|^2 = 1$ and the linearized numerical scheme for the MEW equation is unconditionally stable.

3.2 Stability of Cubic B-Spline GalerkinMethod with Quadratic B-Spline as a Weight Function

A typical member of the matrix system (2.10) can be written in terms of the nodal parameters σ_m^n as

$$l_1\sigma_{m-2}^{n+1} + l_2\sigma_{m-1}^{n+1} + l_3\sigma_m^{n+1} + l_4\sigma_{m+1}^{n+1} + l_5\sigma_{m+2}^{n+1} + l_6\sigma_{m+3}^{n+1} = l_6\sigma_{m-2}^n + l_5\sigma_{m-1}^n + l_4\sigma_m^n + l_3\sigma_{m+1}^n + l_2\sigma_{m+2}^n + l_1\sigma_{m+3}^n$$

where ,

$$l_1 = \frac{1}{60} - \frac{1}{2}\beta - \frac{\lambda\Delta t}{20}, \quad l_2 = \frac{57}{60} - \frac{9}{2}\beta - \frac{25\lambda\Delta t}{20},$$
$$l_3 = \frac{302}{60} + \frac{10}{2}\beta - \frac{40\lambda\Delta t}{20}, \quad l_4 = \frac{302}{60} + \frac{10}{2}\beta + \frac{40\lambda\Delta t}{20},$$
$$l_5 = \frac{57}{60} - \frac{9}{2}\beta + \frac{25\lambda\Delta t}{20}, \quad l_6 = \frac{1}{60} - \frac{1}{2}\beta + \frac{\lambda\Delta t}{20},$$

Substitution of$\sigma_m^n = \ddot{Y}_{19}^n e^{i\beta m h}$, leads to

$$\ddot{Y}_{19}\{l_1 e^{-2i\beta h} + l_2 e^{-i\beta h} + l_3 + l_4 e^{i\beta h} + l_5 e^{2i\beta h} + l_6 e^{3i\beta h}\} = l_1 e^{-2i\beta h} + l_2 e^{-i\beta h} + l_3 + l_4 e^{i\beta h} + l_5 e^{2i\beta h} + l_6 e^{3i\beta h}.$$

simplifying the above equation, we get

$$\ddot{Y}_{19} = \frac{A_{11} - iB_{11}}{A_{11} + iB_{11}},$$

where ,

$$A_{11} = (302 + 300\beta)\cos\left(\frac{\theta}{2}\right)h + (57 - 270\beta)\cos\left(\frac{3\theta}{2}\right)h + (1 - 30\beta)\cos\left(\frac{5\theta}{2}\right)h,$$

$$B_{11} = (120\lambda\Delta t)\sin\left(\frac{\theta}{2}\right)h + (75\lambda\Delta t)\sin\left(\frac{3\theta}{2}\right)h + 3\lambda\Delta t\sin\left(\frac{5\theta}{2}\right)h$$

after simplification, we obtain that $|\ddot{Y}_{19}| = 1$ and the linearized numerical scheme for the MEW equation is unconditionally stable.

3.3 Stability of Quartic B-Spline GalerkinMethod with Cubic B-Spline as a Weight Function

The linearized form of proposed scheme (2.15) takes the form

$$n_1\rho_{m-3}^{n+1} + n_2\rho_{m-2}^{n+1} + n_3\rho_{m-1}^{n+1} + n_4\rho_m^{n+1} + n_5\rho_{m+1}^{n+1} + n_6\rho_{m+2}^{n+1} + n_7\rho_{m+3}^{n+1} + n_8\rho_{m+4}^{n+1} = n_8\sigma_{m-3}^n + n_7\sigma_{m-2}^n + n_6\sigma_{m-1}^n + n_5\sigma_m^n + n_4\sigma_{m+1}^n + n_3\sigma_{m+2}^n + n_2\sigma_{m+3}^n + n_1\sigma_{m+4}^n.$$

where

$$n_1 = \frac{1}{280} - \frac{1}{5}\beta - \frac{\lambda\Delta t}{70}, \quad n_2 = \frac{247}{280} - \frac{55}{5}\beta - \frac{119\lambda\Delta t}{70},$$
$$n_3 = \frac{4293}{280} - \frac{189}{5}\beta - \frac{1071\lambda\Delta t}{70}, \quad n_4 = \frac{15619}{280} + \frac{245}{5}\beta - \frac{1225\lambda\Delta t}{70},$$

$n_5 = \frac{15619}{280} + \frac{245}{5}\theta + \frac{1225\lambda\Delta t}{70}, n_6 = \frac{4293}{280} - \frac{189}{5}\theta + \frac{1071\lambda\Delta t}{20}$,
$n_7 = \frac{247}{280} - \frac{55}{5}\theta + \frac{119\lambda\Delta t}{70}, \quad n_8 = \frac{1}{280} - \frac{1}{5}\theta + \frac{\lambda\Delta t}{70}$,

substitution of $\rho_m^n = \ddot{Y}_{20}^n e^{i\beta m h}$, leads to

$\ddot{Y}_3\{n_1 e^{-3i\beta h} + n_2 e^{-2i\beta h} + n_3 e^{-1i\beta h} + n_4 + n_5 e^{i\beta h} +$ n6e2iβh+n7e3iβh+n8e4iβh=n8e−3iβh+n7e−2iβh+ $n_6 e^{-i\beta h} + n_5 + n_4 e^{i\beta h} + n_3 e^{2i\beta h} + n_2 e^{3i\beta h} + n_1 e^{4i\beta h}$.

Simplifying the above equation, weget

$\ddot{Y}_{20} = \frac{A_{12} - iB_{12}}{C_{12} + iD_{12}}$,

where ,

$A_{12} = (\frac{15619}{280} + \frac{245}{5}\theta + \frac{1225\lambda\Delta t}{70}) + (\frac{19912}{280} + \frac{56}{5}\theta - \frac{154\lambda\Delta t}{70})\cos(\beta h) + (\frac{4540}{280} - \frac{244}{5}\theta - \frac{952\lambda\Delta t}{70})\cos(2\beta h) + (\frac{248}{280} - \frac{56}{5}\theta - \frac{118\lambda\Delta t}{70})\cos(3\beta h) + (\frac{1}{280} - \frac{1}{5}\theta - \frac{\lambda\Delta t}{70})\cos(4\beta h)$,

$B_{12} = \left(\frac{11326}{280} + \frac{434}{5}\theta - \frac{2296\lambda\Delta t}{70}\right)\sin(\beta h) + \left(\frac{4046}{280} - \frac{134}{5}\theta -\right.$ 1190λΔt70sin2βh+(246280−5456−120λΔt70)sin3βh +(1280−156−λΔt70)sin(4βh),

$C_{12} = (\frac{15619}{280} + \frac{245}{5}\theta - \frac{1225\lambda\Delta t}{70}) + (\frac{19912}{280} + \frac{56}{5}\theta + \frac{154\lambda\Delta t}{70})\cos(\beta h) + (\frac{4540}{280} - \frac{244}{5}\theta + \frac{952\lambda\Delta t}{70})\cos(2\beta h) +$ (248280−5656+118λΔt70)cos3βh+(1280−156+λΔt70)sin(4βh),

$D_{12} = (\frac{11326}{280} + \frac{434}{5}\theta + \frac{229\lambda\Delta t}{70})\sin(\beta h) + (\frac{4046}{280} - \frac{134}{5}\theta + \frac{1190\lambda\Delta t}{70})\sin(2\beta h) + (\frac{246}{280} - \frac{54}{5}\theta + \frac{120\lambda\Delta t}{70})\sin(3\beta h) + (\frac{1}{280} - \frac{1}{5}\theta + \frac{\lambda\Delta t}{70})\sin(4\beta h)$.

after simplification, we obtain that $|\ddot{Y}_{20}| = 1$ and the linearized numerical scheme for the MEW equation is unconditionally stable.

3.4 Stability of Quintic B-Spline GalerkinMethod with Quartic B-Spline as a Weight Function

The linearized form of proposed scheme (2.20) takes the form

$$q_1\vartheta_{m-3}^{n+1} + q_2\vartheta_{m-2}^{n+1} + q_3\vartheta_{m-1}^{n+1} + q_4\vartheta_m^{n+1} + q_5\vartheta_{m+1}^{n+1} + q_6\vartheta_{m+2}^{n+1} + q_7\vartheta_{m+3}^{n+1} + q_8\vartheta_{m+4}^{n+1} + q_9\vartheta_{m+5}^{n+1} + q_{10}\vartheta_{m+6}^{n+1} = q_{10}\vartheta_{m-3}^{n} + q_9\vartheta_{m-2}^{n} + q_8\vartheta_{m-1}^{n} + q_7\vartheta_m^{n} + q_6\vartheta_{m+1}^{n} + q_5\vartheta_{m+2}^{n} + q_4\vartheta_{m+3}^{n} + q_3\vartheta_{m+4}^{n} + q_2\vartheta_{m+5}^{n} + q_1\vartheta_{m+6}^{n}.$$

where

$q_1 = \frac{1}{1260} - \frac{1}{14}\theta - \frac{\lambda\Delta t}{252}, \quad q_2 = \frac{1013}{1260} - \frac{245}{14}\theta - \frac{501\lambda\Delta t}{252}$,
$q_3 = \frac{47840}{1260} - \frac{3800}{14}\theta - \frac{14106\lambda\Delta t}{252}, \quad q_4 = \frac{455192}{1260} - \frac{7280}{14}\theta - \frac{73626\lambda\Delta t}{252}$
$q_5 = \frac{13103540}{1260} + \frac{11326}{14}\theta - \frac{67956\lambda\Delta t}{252}, \quad q_6 = \frac{13103540}{1260} + \frac{11326}{14}\theta + \frac{67956\lambda\Delta t}{252}$,
$q_7 = \frac{455192}{1260} - \frac{7280}{14}\theta + \frac{73626\lambda\Delta t}{252}, q_8 = \frac{47840}{1260} - \frac{3800}{14}\theta + \frac{14106\lambda\Delta t}{252}$,
$q_9 = \frac{1013}{1260} - \frac{245}{14}\theta + \frac{501\lambda\Delta t}{252}, \quad q_{10} = \frac{1}{1260} - \frac{1}{14}\theta + \frac{\lambda\Delta t}{252}$,

the error in typical mode of amplitude $\ddot{Y}_{21}^n$,

$\vartheta_m^n = \ddot{Y}_{21}^n e^{i\beta m h}$,

substituting the above Fourier mode into linearized form gives

$\ddot{Y}_{21}^{n+1} = g_{13}\ddot{Y}_{17}^n$,

the growth factor g_{13} has the form:

$$g_{13} = \frac{b_1 e^{5i\beta h} + (b_2 + b_{10})(e^{4i\beta h} + e^{-4i\beta h}) + (b_3 + b_9)(e^{3i\beta h} + e^{-3i\beta h}) + (b_4 + b_8)(e^{2i\beta h} + e^{-2i\beta h}) + (b_5 + b_7)(e^{i\beta h} + e^{-i\beta h}) + (b_6}{b_{10} e^{5i\beta h} + (b_9 + b_1)(e^{4i\beta h} + e^{-4i\beta h}) + (b_8 + b_2)(e^{3i\beta h} + e^{-3i\beta h}) + (b_7 + b_3)(e^{2i\beta h} + e^{-2i\beta h}) + (b_6 + b_4)(e^{i\beta h} + e^{-i\beta h}) + b_5}$$

So that the magnitude of the growth factor $|\ddot{Y}_{21}| \leq 1$, and the linearized recurrence relation based on the present scheme is unconditionally stable.

3.5 Stability of Sextic B-Spline GalerkinMethod with Quintic B-Spline as a Weight Function

The linearized form of proposed scheme (2.25) takes the form

$$p_1\tau_{m-3}^{n+1} + p_2\tau_{m-2}^{n+1} + p_3\tau_{m-1}^{n+1} + p_4\tau_m^{n+1} + p_5\tau_{m+1}^{n+1} + p_6\tau_{m+2}^{n+1} + p_7\tau_{m+3}^{n+1} + p_8\tau_{m+4}^{n+1} + p_9\tau_{m+5}^{n+1} + p_{10}\tau_{m+6}^{n+1} + p_{11}\tau_{m+7}^{n+1} + p_{12}\tau_{m+8}^{n+1} = p_{12}\tau_{m-3}^{n} + p_{11}\tau_{m-2}^{n} + p_{10}\tau_{m-1}^{n} + p_9\tau_m^{n} + p_8\tau_{m+1}^{n} + p_7\tau_{m+2}^{n} + p_6\tau_{m+3}^{n} + p_5\tau_{m+4}^{n} + p_4\tau_{m+5}^{n} + p_3\tau_{m+6}^{n} + p_2\tau_{m+7}^{n} + p_1\tau_{m+8}^{n}.$$

where

$p_1 = \frac{1}{5544} - \frac{1}{42}\theta - \frac{\lambda\Delta t}{462}, \quad p_2 = \frac{4083}{5544} - \frac{1011}{42}\theta - \frac{2035\lambda\Delta t}{462}$,
$p_3 = \frac{478271}{5544} - \frac{45815}{42}\theta - \frac{150601\lambda\Delta t}{462}, \quad p_4 = \frac{10187685}{5544} - \frac{360525}{42}\theta - \frac{2050851\lambda\Delta t}{462}$,
$p_5 = \frac{66318474}{5544} - \frac{447810}{42}\theta - \frac{7534626\lambda\Delta t}{462}, \quad p_6 = \frac{162512286}{5544} + \frac{855162}{42}\theta - \frac{5986134\lambda\Delta t}{462}$,
$p_7 = \frac{162512286}{5544} + \frac{855162}{42}\theta + \frac{5986134\lambda\Delta t}{462}, p_8 = \frac{66318474}{5544} - \frac{447810}{42}\theta + \frac{7534626\lambda\Delta t}{462}$,
$p_9 = \frac{10187685}{5544} - \frac{-360525}{42}\theta + \frac{2050851\lambda\Delta t}{462}, \quad p_{10} = \frac{478271}{5544} - \frac{45815}{42}\theta + \frac{150601\lambda\Delta t}{462}$,
$p_{11} = \frac{4083}{5544} - \frac{1011}{42}\theta + \frac{2035\lambda\Delta t}{462}, p_{12} = \frac{1}{5544} - \frac{1}{42}\theta + \frac{\lambda\Delta t}{462}$

the error in typical mode of amplitude $\ddot{Y}_{22}^n$,

$\rho_m^n = \ddot{Y}_{22}^n e^{i\beta m h}$,

substituting the above Fourier mode into linearized form gives

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$\ddot{Y}_{22}^{n+1} = g_{14}\ddot{Y}_{22}^{n}$,

the growth factor g_{14} has the form:

$$g_{14} = \frac{b_1 e^{6i\beta h} + (b_2 + b_{12})(e^{5i\beta h} + e^{-5i\beta h}) +}{b_{12} e^{6i\beta h} + (b_{11} + b_1)(e^{5i\beta h} + e^{-5i\beta h}) +}$$

$$\frac{(b_3 + b_{11})(e^{4i\beta h} + e^{-4i\beta h}) + (b_4 + b_{10})(e^{3i\beta h} + e^{-3i\beta h})}{(b_{10} + b_2)(e^{4i\beta h} + e^{-4i\beta h}) + (b_9 + b_3)(e^{3i\beta h} + e^{-3i\beta h})}$$

$$\frac{+ (b_5 + b_9)(e^{2i\beta h} + e^{-2i\beta h}) + (b_6 + b_8)(e^{i\beta h} + e^{-i\beta h}) + b_7}{+ (b_8 + b_4)(e^{2i\beta h} + e^{-2i\beta h}) + (b_7 + b_5)(e^{i\beta h} + e^{-i\beta h}) + b_6}$$

So that the magnitude of the growth factor $\left|\ddot{Y}_{22}\right| \leq 1$, and the linearized recurrence relation based on the present scheme is unconditionally stable.

IV. Numerical Example and Results

A numerical algorithm for the solutions of the MEW equation should describes adequately the motion of a single solitary wave and should exhibit the same conservation laws as the differential equation. The numerical algorithms set up in Section 2 is validated for the MEW equation by following the motion of a single solitary wave across the mesh. It is expected that Eq.(1.2a) will represent not only the solitary wave solution for an unbounded region but also the solitary wave solution for bounded region of sufficient size. ThusEq.(1.2a) is used as initial condition with range $0 \leq x \leq 80$, space step $h = 0.1$, time step $\Delta t = 0.05$, amplitude $A = 0.25$,$\mu = 1$and $x_0 = 30$. The simulation is run to time $t = 20$ and the quantities I_1, I_2, I_3 are calculated from the sums

$$I_1 = h\sum_{J=1}^{N} U_J^n,$$

$$I_2 = h\sum_{J=1}^{N} (U_J^n)^2 + \mu((U_x)_J^n)^2,$$

$$I_3 = h\sum_{J=1}^{N} (U_J^n)^4,$$

where U_J^n and $(U_x)_J^n$ are mesh values of the numerical solution for the simulation region $0 \leq x \leq x_N$ and the error norm $L_2$and L_∞ are recorded throughout where

$$L_2 = \left\|U - U_N\right\|_2 \cong \sqrt{h\sum_{J=0}^{N}\left|U_j - U_N\right|^2}$$

and

$$L_\infty = \left\|U - U_N\right\|_\infty \cong \max_J \left|U_j - U_N\right|,$$

initial condition Eq. (1.1c) enables the integrals (1.3) to be determined analytically as[8]

$$C_1 = \frac{\Tau\pi}{\mathcal{K}_1}, \qquad C_2 = \frac{2\Tau^2}{\mathcal{K}_1} + \frac{2\mu\mathcal{K}_1\Tau^2}{3}, \qquad C_3 = \frac{4\Tau^2}{3\mathcal{K}_1}.$$

Table 1: Invariants and error norms for Galerkin B-spline methods with different weight function

The Methods	I_1	I_2	I_3	L_2	L_∞
Quadratic with linear	0.7854	0.1250	0.0052	1.9902e-015	5.8287e-016
Cubic with Quadratic	0.7854	0.1251	0.0052	9.3159e-004	2.0820e-004
Quartic with Cubic	0.7854	0.1260	0.0052	3.5741e-005	8.6745e-006
Quintic with Quartic	0.7854	0.1258	0.0052	1.0396e-005	2.7161e-006
Sextic with Quintic	0.7854	0.1257	0.0052	3.5916e-005	8.7144e-006

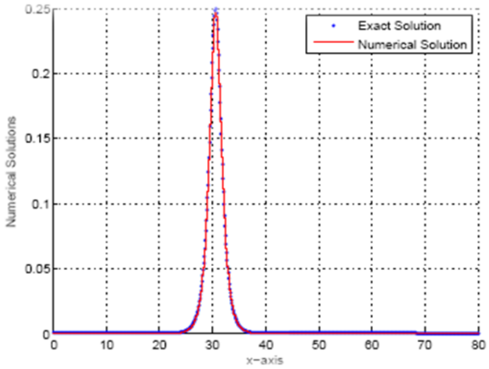

Fig.1. The motion of a single solitary wave with $h = 0.1$and $\Delta t = 0.05$at$t = 0$ to 20

V. Conclusions

The B-spline weighted residual methods based on Galerkinsuccessfully models the motion and interaction of solitary waves of the MEW equation. Several cases are chosen from literature to validate performance of the proposed methods. The accuracy of these methodsare checked through $L_2$and L_∞error norm and the invariants C_1, C_2 and C_3. It has been observed that the error where sufficiently small and the invariants are almost kept constant during simulation.From Table (1) it is seen that conservation is excellent since throughout the simulationI_2varies from the analytic value of $C_2 = 0.16667$,$I_1$is constant at$C_1 = 0.7854$, and I_3 is also constant at $C_3 = 0.00521$ the total error, measured by the L_2 error norm, and the maximum error measured by the L_∞ error norm. The properties required of a good numerical schemes described above are clearly exhibited.The results obtained from numerical experiments are in agreement with some earlier results available in the literature. Linear stability analysis proved that the previous methods are unconditionally stable theoretically and this has been supported by the test problem as well, the simulation process is made by using MATLAB 2011 software package.

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