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Optimal Energy Control Strategy of PHEV Based on the Pontryagin's Minimum Principle Algorithm

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***Abstract* – In order to improve the fuel economy of plug-in hybrid electric vehicles (PHEVs), the energy control strategy of the whole vehicle is optimized. Firstly, the optimization methods of different energy control strategies are analyzed, and the Pontryagin's Minimum Principle (PMP) and the equivalent fuel consumption theory are selected as the optimal control algorithm. Secondly, the configuration of PHEV and the research objectives of the power control system are determined. Thirdly, the energy control problem is analyzed by the PMP theory, and the improvement measures for the energy control problem are put forward by the minimum control theory of the equivalent fuel consumption. Fourthly, after analyzing the relationship between the equivalent fuel factor and reference SOC, the equivalent minimum fuel consumption strategy (ECMS) energy control model is established by MATLAB/Simulink. Finally, combined with Cruise software, the PHEV simulation model is simulated, and the simulation results are analyzed. The results show that compared with the CD/CS energy control strategy, the ECMS energy control strategy reduced the 100 km fuel consumption of the vehicle by 8.93% under 6 times NEDC driving conditions.**

***Keywords* – Plug-in Hybrid Electric Vehicle, CD/CS, PMP, ECMS, MATLAB/Simulink, Cruise.**

I. Introduction

PHEV power system is a very complex nonlinear power system [1]. Compared with the traditional HEV control strategy, the PHEV has a large power battery which can obtain electricity from the power grid through an external charger. Therefore, the designed energy control strategy needs to improve the efficiency of the engine and electrical machinery, and maximize the consumption of electric energy. At present, energy control strategies can generally be divided into four types: logic threshold energy control strategy [2], global optimization energy control strategy [3], instantaneous optimization energy control strategy [4], and intelligent control optimization energy control strategy [5].

The logic threshold control strategy is mainly designed according to the designer's engineering experience and the working characteristics of each key power component. The hybrid power system is divided into different working modes by a series of threshold values. It has the advantages of simple design, fast response, good robustness and so on [6]. H. Banvait et al. [7] designed the vehicle controller using the fixed logic threshold, in order to realize the distribution of vehicle torque and the switching. However, the logic threshold control strategy poorly adapts of working conditions, cannot guarantee the optimal fuel economy of PHEV under changing working conditions.

The global optimization energy control strategy is a reverse optimization control strategy, which can make the vehicle distribute energy optimally under a given driving condition. Moura et al. [8] analyzed the relationship among the driving mileage, battery SOC and global energy control strategy, and used the random dynamic algorithm to design global optimized energy control strategy, so as to reasonably distribute the power output of each power component. Liu et al. [9] used back propagation neural network to build the dynamic model, and verify the proposed heuristic DP on-line energy control strategy by the measured driving conditions. The results show that compared with the off-line global optimal energy control strategy, the proposed online strategy reduces fuel consumption by 4%. However, the DP algorithm needs to know the driving conditions and the reverse solution requires a lot of complex operations. Therefore, the global optimized energy control strategy cannot be applied to the real vehicle for these two reasons, and can only be used as a theoretical optimal solution [10].

The instantaneous optimization energy control strategy is a forward optimization control strategy, which can enable vehicles to distribute energy optimally under given driving conditions. Compared with the energy control strategy based on DP algorithm, the energy control strategy based on PMP algorithm is a typical instantaneous optimization energy control strategy, has the advantage of less computation, is expected to meet the real-time requirements through some improvement. Ouddah et al. [11] used the PMP algorithm to obtain the bus's optimal energy control strategy based on fixed line in off-line state, and to realize the online application of energy control strategy through SOC feedback regulation. Whether DP or PMP algorithm is used, the working condition information needs to be predefined. However, mileage, future speed and other information are difficult to obtain accurately. Therefore, the above optimization algorithm is difficult to obtain practical.

Intelligent control energy control strategy uses the artificial neural network algorithm, genetic algorithm, particle swarm optimization and other intelligent control algorithms to optimize the PHEV energy allocation, so as to achieve optimal control. Kavya P Divakarla et al. [12] used artificial neural network to design the vehicle controller and train its control precision and robustness under various driving conditions such as urban working conditions and high speed working conditions, so that the designed vehicle controller can automatically adapt to different driving conditions and improve the fuel economy of the vehicle. Chen et al. [13] used the particle swarm algorithm to optimize four thresholds of the proposed three rule-based strategies and achieve the efficient utilization of vehicle energy. But the intelligent control energy control strategy often needs a lot of data to train the model, so that the result of its optimization often depends on the training data.

The energy control strategy has developed from single goal and single system to multi-objective integration optimization, information sharing of multi-system, and from rule-based control strategy to intelligent control strategy. As shown in the above research, the most logical threshold energy strategies are designed based on the working characteristics of the engine, the electrical machinery and other power components [14]. The optimized energy control strategy is mainly to regard the energy allocation problem of power system as mathematical problem with constraint, and then the logical threshold control strategy is continuously improved by the corresponding optimization control theory, so as to overcome the shortcomings of logical threshold control strategy and improve the fuel economy under different driving conditions. At the same time, the control results are formed into a simple MAP diagram which is obtained by the optimization algorithm, and the control rules can also be used to control the power system quickly and effectively. Therefore, there is no strict standard for the division of energy control strategies, but only different forms of energy control strategies. The different types of control strategies can be converted to each other [15].

The article is organized as follows. Section II determines the objectives and methods of the PHEV optimal control and uses PMP theory to solve the optimization problem of energy control. In Section III, the ECMS optimized energy control strategy is designed according to the PMP optimal algorithm, and the ECMS energy control model is established. In Section IV, the simulation and test validation is implemented. Finally, Section V concludes this article.

II. Energy Control Problem Solving and Optimization

According to the power transfer mode and the layout structure of each power component, the PHEV configuration can generally be divided into three types: series type, parallel type and mixed type. This paper uses the PHEV of coaxial parallel P2 configuration as the research object, and its structure is shown in Fig.1. The engine and the electrical machinery can be connected to the drive shaft through a mechanical coupling device and output a certain proportion of power. In the process of automobile braking, the electrical machinery can convert some kinetic energy into electric energy for recovery. Because the engine and electrical machinery can drive the car alone or together, the configuration can adopt smaller power components, such as lower capacity battery and smaller electrical machinery, which greatly reduces the quality and cost of the whole vehicle. However, because of the mechanical connection between the engine and the drive wheel in the configuration, the working efficiency of the engine cannot be brought into full play when the driving condition changes. The basic structural parameters of the vehicle are shown in Table 1.

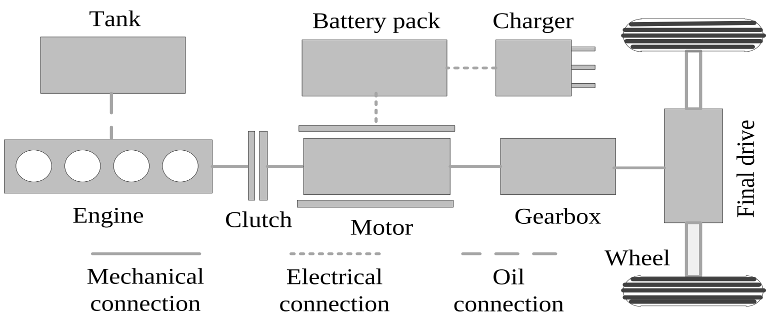

Fig. 1. Parallel PHEV power system structure.

Table 1. Basic structure parameters of PHEV.

Type	Parameter name (unit)	Parameter value
Vehicle	Mass (kg)	1449
	Front and rear axle load (kg)	873/675
	Wheelbase (mm)	3600
	Rolling radius（m）	0.3
	Front area A（m^2）	2.25
	Drag coefficient C_D	0.32
	Rolling resistance coefficient f	0.015
	Mechanical transmission efficiency η	0.95
Engine	Power（kW）	83
	Peak speed（rpm）	6000
	Peak torque（Nm）	170/4000

Type	Parameter name (unit)	Parameter value
Motor	Peak torque（Nm）	140
	The maximum power（kW）	30
Battery	Capacity（Ah）	35
	Electric energy（kWh）	8.4
	Voltage level（V）	300
Gearbox	Ratio	3.85/2.44/1.71/1.27/1.00/0.82/0.70
Final drive	Ratio	3.94

A. *Determination of Control Target of Power System and Solving Method*

The optimization objects of energy control strategy are various, such as fuel consumption, exhaust emission and operating cost [16]. The paper selects the minimum equivalent fuel consumption of the PHEV as the objective function. The state variable x represents battery SOC, and its main constraints include the state transfer equation of the power system and the working range of the SOC. The control variable u is the electrical machinery output torque and its main constraints are generally the output torque and power range of engine and electrical machinery, and the parameters such as the voltage, current, power, SOC working range of the power battery. Therefore, the expression of PHEV optimal control problem is [17]:

$$\begin{cases} \min : J \int_{0}^{tf} \dot{m}_{eqv} \ (x,u,t)dt \\ s.t.u \ \in U \\ \dot{x} = f(x,u,t) \\ x(0) = x_0 \\ x(t_f)x_f \end{cases} \tag{1}$$

At present, there are three kinds of solutions to the optimal control problem of PHEV: dynamic programming, Pontryagin minimum principle and minimum equivalent fuel consumption.

The dynamic programming mainly solves the problem of multi-stage decision optimization, which belongs to the category of global optimization. For optimization problem of the PHEV energy control strategy, the state variables of DP energy control strategy are generally battery SOC value, transmission ratio, and the control variables are generally engine output torque, electrical machinery output torque, transmission gear. On the premise of known vehicle driving condition information, the continuous variables of the power system are discretized by DP theory. Taking the minimum fuel consumption or exhaust emission of the whole vehicle as the optimization objective function, the optimization objective function is solved by reverse solution method, so as to realize the global optimal distribution of the vehicle power in the whole driving process. The simulation results show that the DP control method can achieve the best fuel consumption and exhaust emission performance [18-19]. However, the algorithm has the disadvantages of complex calculation process, and cannot carry out real-time control. Therefore, it only makes the control strategy have good control effect under the premise of known driving condition information. At the same time, the control results of the algorithm can be used as a reference standard for designing other control strategies.

The Pontryagin's minimum principle belongs to a global optimization control algorithm. It mainly introduces Lagrange function to establish a new Hamilton equation which considers the constraints in the performance indicator function. Thus, the minimum value of performance index function can be obtained by solving the new Hamilton equation [20]. Although the algorithm has good analytical properties, the solution is more complicated.

The ECMS energy control strategy belongs to an instantaneous optimization energy control strategy based on heuristic experience. The electric energy consumption of the electrical machinery is equivalent to the fuel consumption of the engine by the equivalent factor in the hybrid power system. Then, the minimum objective function of fuel consumption is established. The optimal torque distribution between the engine and the electrical machinery is obtained by solving the minimum equivalent consumption function [21]. Musardo et al. [22] first selected the oil-electric conversion coefficient according to the energy loss of charging and discharging of the power battery after the end of the vehicle driving condition, and then update and adjust the oil-electric conversion coefficient according to the historical driving information and prediction information of the vehicle, so as to ensure the optimal distribution of power. In order to obtain the optimal equivalent factor, Zeng et al. [23] optimized the equivalent factor by particle swarm algorithm, obtained the equivalent factor MAP of the SOC and mileage, and made the algorithm have a good control effect. Because the ECMS energy control strategy does not need to know the driving condition of the vehicle and its control effect is very close to the optimal global optimization control effect, it can ensure the optimal distribution of the vehicle power output at each time and realize real-time control of vehicles. Therefore, this method has been widely studied and paid attention to in hybrid vehicles.

According to the above analysis, compared with dynamic programming, the ECMS energy control strategy has less computation, can be used for real-time control, and can obtain almost the same fuel economy. Meanwhile, its control effect is very close to that of dynamic programming. ECMS control algorithm can be approximated to the PMP control algorithm if the voltage and internal resistance of battery do not change with the SOC of the battery. The ECMS objective function is solved by solving Hamiltonian function method [24]. Because the selection of equivalent factors determines the control effect of ECMS energy control strategy, and the equivalent factors in ECMS strategy are equivalent to the cooperative state variables in PMP algorithm, this paper selects the principle of PMP to design ECMS [25].

B. *The Minimum Optimization Principle of Pontryagin's Minimum Principle*

Pontryagin's Minimum Principle (PMP) is also known as the Pontryagin's maxima principle, and is the optimal control theory which is proposed by the former Soviet scholar. It is mainly used to solve continuous or discontinuous optimal control problems, and is an important part of modern control theory to solve optimal control problems after replacing the classical variational method [26]. PMP principle proposes that a necessary condition for a performance index function to reach the minimum is that the optimal control decision $u^*(t)$ minimizes the value of the Hamiltonian function $H[x^*(t), \lambda^*(t), u^*(t), t]$ [27].

- According to the mathematical model and cost function of the control system, the Hamilton function is established, and the necessary conditions and the boundary conditions for obtaining the optimal solution of the Hamilton function are listed.

- When the $H(x, \lambda, u, t)$ is minimum, the candidate optimal control decision $u' = u(x, \lambda, t)$ is obtained.

- The necessary conditions of the optimal solution are obtained by optimal control decision u'. The optimal trajectory x^* of state variables and the optimal trajectory λ^* of cooperative variables are obtained according to boundary conditions. The optimal trajectory of two cooperative variables is brought into the optimal control decision $u^* = u(x^*, \lambda^*, t)$, and the optimal control decision of the research object is obtained.

C. *Energy Control Model Based on PMP*

On the premise of the PHEV dynamics, the whole vehicle power system is considered as a complex nonlinear system, and the minimum fuel consumption problem is equivalent to the constrained optimal control problem.

C.1. *Performance Indicator Function*

The indicator function can be the life of battery, fuel consumption, exhaust emission, or new goals which are combined with the above multiple objectives. In this paper, the fuel consumption of the whole vehicle is chosen as the objective function, which makes the fuel consumption of the whole vehicle minimize under certain working conditions. The PMP algorithm can optimize the objective functions of exhaust emission, battery SOC [29] [30]. In this paper, the minimum fuel consumption is taken as the only optimization objective of the optimal energy control strategy. The performance of the vehicle is as follows:

where, T_e and n_e are the torque and speed of engine, D is the feasible area for engine work, $\dot{m}(u(t)t)$ fuel consumption of power systems at every moment along the optimal operating curve, t_f is the simulation end time, $u(t)$ is the electrical machinery output torque at every moment.

C.2. *State and Control Variables*

Battery SOC is selected as the state variable of the whole vehicle power system, and the SOC estimation method is not used as the focus of study here. The state variable of the vehicle power system is as follows.

$$x(t) = SOC(t) = SOC(t_0) - \int_{t_0}^{t} \frac{I_{batt}(\tau)}{Q_{batt}} d\tau \tag{2}$$

where, Q_{batt} is the battery capacity, $I_{batt}(\tau)$ is the battery current at every moment.

In the PHEV system, the paper selects the control variable $u(t)$ of the control power system as the electrical machinery output torque at every moment. According to the PMP theory, the state equation of the whole vehicle power system is:

$$\dot{x}(t) = f(x(t), u(t), t) \tag{3}$$

Because the internal resistance of the power consumes energy, the output power of PHEV battery follows the output power formula of battery.

$$P_{batt} = V_{oc}(SOC) \cdot I_{batt} - R_0(SOC) \cdot I_{batt}^{\,2} \tag{4}$$

where, V_{oc} is the open circuit voltage of batteries, I_{batt} is the current of batteries, R_0 is the equivalent internal resistance of batteries.

When $P_{batt} > 0$ and $I_{batt} > 0$, the battery outputs electric energy to the electrical machinery and the electrical machinery works as the motor. When $P_{batt} < 0$ and $I_{batt} < 0$, the engine provides power for the electrical machinery, and the electrical machinery is used to charge the battery. The relationship between battery and electrical machinery is as follows.

$$P_{batt} = \begin{cases} \eta_{em} \cdot P_{em} & P_{em} < 0 \\ \dfrac{P_{em}}{\eta_{em}} & P_{em} \geq 0 \end{cases} \quad (5)$$

where, P_{em} is the power of the electrical machinery; η_{em} is the efficiency of the electrical machinery.

The output current of the battery is as follows which is derived by (4) and (5).

$$I_{batt} = \frac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4 \cdot P_{batt}(T_m) \cdot R_o(SOC)}}{2R_0(SOC)} \quad (6)$$

where, R_0 is the internal resistance of battery.

According to the ampere metrology, the equivalent charge and discharge capacity of battery is as follows.

$$SOC = \begin{cases} SOC_0 - \int_{t_0}^{t_f} \dfrac{\eta_{batt} \cdot I}{Q_{batt}} dt, \text{discharging} \\ SOC_0 + \int_{t_0}^{t_f} \dfrac{I}{\eta_{batt} \cdot Q_{batt}} dt, \text{charging} \end{cases} \quad (7)$$

Where, Q_{batt} is the power of the battery pack when it is discharged a constant current I, η_{batt} is the charge/discharge efficiency of the battery.

By conducting the derivation of the discharge time on both sides of (7), the rate of change of the SOC is the ratio of the battery to rated capacity.

$$SOC = \begin{cases} SOC_0 - \int_{t_0}^{t_f} \dfrac{\eta_{batt} \cdot I}{Q_{batt}} dt, \text{discharging} \\ SOC_0 + \int_{t_0}^{t_f} \dfrac{I}{\eta_{batt} \cdot Q_{batt}} dt, \text{charging} \end{cases} \quad (8)$$

where, Q_{batt} is the rated capacitance of battery.

The state transfer equation of battery is derived by (6) and (8).

$$\dot{x}(t) = S\dot{O}C = \begin{cases} -\dfrac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4 \cdot P_b(T_m) \cdot R_o(SOC)}}{2R_0(SOC) \cdot Q_{batt}} \cdot \eta_{batt}, \text{ discharging} \\ \dfrac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4 \cdot P_b(T_m) \cdot R_o(SOC)}}{2R_0(SOC) \cdot Q_{batt} \cdot \eta_{batt}}, \text{ charging} \end{cases} \quad (9)$$

C.3. *Constraints*

In order to prevent the SOC below the threshold for switching CS mode at the end of the vehicle, the minimum threshold value of the battery SOC is set to be 0.4. Because the internal characteristics of the motor are not studied, the working performance of the motor can be regarded as symmetrical during driving and braking. According to the above analysis, the global optimization objectives and constraints of the PHEV energy control strategy are as follows under the PMP algorithm.

$$\begin{aligned} &\min J = \int_{t_0}^{t_f} \dot{m}_f(u(t))dt \\ &SOC \in [SOC_{\min}, SOC_{\max}] \\ &SOC(t_f) \geq 0.4 \\ &P_{batt} \in [P_{batt\_\min}, P_{batt\_\max}] \\ &T_m \in [T_{m\min}, T_{m\max}] \\ &T_e \in [T_{e\min}, T_{e\max}] \end{aligned} \tag{10}$$

The global optimization problem of the energy control is transformed into several instantaneous optimization problems with Hamilton function by introducing the cooperative state quantity. The Hamilton function is equal to the sum of the engine instantaneous fuel consumption and product of the synergetic state variable and SOC instantaneous variation [31]. The torque distribution of power system will get the corresponding results by solving the minimum value of Hamilton function at each moment. It should be noted that the initial value of the synergetic state variable will vary with different driving events.

Combined with (3), the Hamiltonian function of PHEVs is established by introducing the synergetic state variable.

$$\mathrm{H}(\mathrm{x(t), u(t)}, \lambda(\mathrm{t}), \mathrm{t}) = \dot{m}_f(u(t),\ t) + \lambda(t)\cdot f(x(t),\ u(t), t) \tag{11}$$

where, $\dot{m}_f(u(t),t)$ is the instantaneous fuel consumption of engines, $\lambda(t)$ is the synergistic state variables which is related to the driving condition, $f(x(t),u(t),t)$ is the system state transfer equation and the SOC instantaneous variation equation of battery.

$$f(x(t),u(t),t) = S\dot{O}C \tag{12}$$

Combined with (11), the Hamiltonian function can be expressed as:

$$\mathrm{H}(\mathrm{x(t), u(t)}, \lambda(\mathrm{t}),\mathrm{t}) = \frac{P_{eng}(t)}{\eta_{eng}(t)} + \lambda(t)\cdot\frac{P_{batt}(\mathrm{t})}{\eta_{batt}} \tag{13}$$

where, $P_{eng}(t)$ the engine's output power, η_{eng} is the efficiency of converting fuel chemical energy into mechanical energy in engines, η_{batt} is the charge-discharge efficiency of batteries, $P_{batt}(t)$ is the output power of battery.

The optimal result $u^*(t)$ of the optimal control variable can be obtained by finding the minimum value of Hamilton function at every moment.

$$u^*(t) = \arg\min H(x(t),u(t),\lambda(t),t)$$

$$= \arg\min H\{\dot{m}_f(u(t),t) + \lambda(t)\cdot f(x(t),u(t),t)\} \tag{14}$$

The synergetic state transfer equation is as follow.

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} \tag{15}$$

State variables and synergetic state variables satisfy the following constraints under the optimal solution of system control variables.

$$x^*(t_0) = x_0 \tag{16}$$

$$x^*(t_f) = x_{target} \tag{17}$$

$$\dot{x}(t) = \left.\frac{\partial H}{\partial \lambda}\right|_{u^*(t)} = f(x^*(t), u^*(t), t) \tag{18}$$

$$\dot{\lambda}^*(t) = \left.\frac{\partial H}{\partial x}\right|_{u^*(t)} = -\frac{\delta \dot{m}_f}{\delta \lambda}(x^*(t), u^*(t)) - \lambda^*(t) \cdot (\frac{\delta f}{\delta \lambda}(x^*(t), u^*(t)) \tag{19}$$

where, $x^*(t_0)$ is the initial state of the state variable, $x^*(t_f)$ is the termination state of the state variable, x_{target} is the target state of the state variable in the termination phase.

Because the instantaneous fuel consumption of the engine is mainly affected by the torque and speed of the engine, but not by the SOC value of battery, the state transfer equation of the control system is as follows.

$$\dot{\lambda}(t) = -\lambda \frac{\partial \dot{x}(t)}{\partial x} = -\lambda(t) \cdot \frac{\partial}{\partial x} \cdot \frac{V_{oc}(x(t)) - \sqrt{V^2{}_{oc}(x(t)) - 4P_{batt}(u(t)) \cdot R_0(x(t))}}{2R_0(x(t)) \cdot Q_{batt}} \tag{20}$$

Therefore:

$$\dot{x}(t) = \frac{\partial H}{\partial \lambda} = -\frac{V_{oc}(SOC) - \sqrt{V^2{}_{oc}(SOC) - 4P_{batt}(T_m) \cdot R_0(SOC)}}{2 \cdot R_0(SOC) \cdot Q_{batt}} \tag{21}$$

Although the Hamiltonian operator has the possibility to use the PMP control strategy for real-time control, it is difficult to obtain the initial value of the synergy state and update the value of the synergy in practice. Therefore, the ECMS energy control strategy is designed according to the necessary conditions which use the PMP control algorithm to solve the optimal solution of the objective function.

III. Optimization Design of the Energy Control Strategy

A. *ECMS Control Theory*

In 1999, the equivalent consumption minimization strategy (ECMS) is proposed by Paganelli et al, and is applied to the energy control strategy of hybrid vehicles. It is a semi-analytical method based on PMP theory, which has advantages of the fast calculation speed, strong real-time performance, compatibility with any configuration HEV and no need for global condition information [32].

The working principle of the ECMS is shown in Fig. 2. The battery is used as a virtual engine. The total fuel consumption is equivalent to the sum of the fuel consumption by the engine and fuel consumption of the virtual engine which is converted from electricity consumed by the vehicle virtual engine [33]. At the stage of electricity consumption, the PHEV adopts the working state of electrical machinery drive and engine as auxiliary. At this time, most of the electricity consumed will be replenished through the power grid in the future, and a small amount of electricity consumed will be replenished through braking energy recovery. At any time in the future and at any given operating point, the battery will have charging and discharging working states, and the energy flow of the PHEV will be converted into equivalent fuel consumption by the equivalent factor. If the electrical machinery charges the battery, the equivalent fuel obtained by the virtual engine is equivalent to being put back into the tank. Here, it is equivalent to saving fuel consumption, and fuel storage rate is negative.

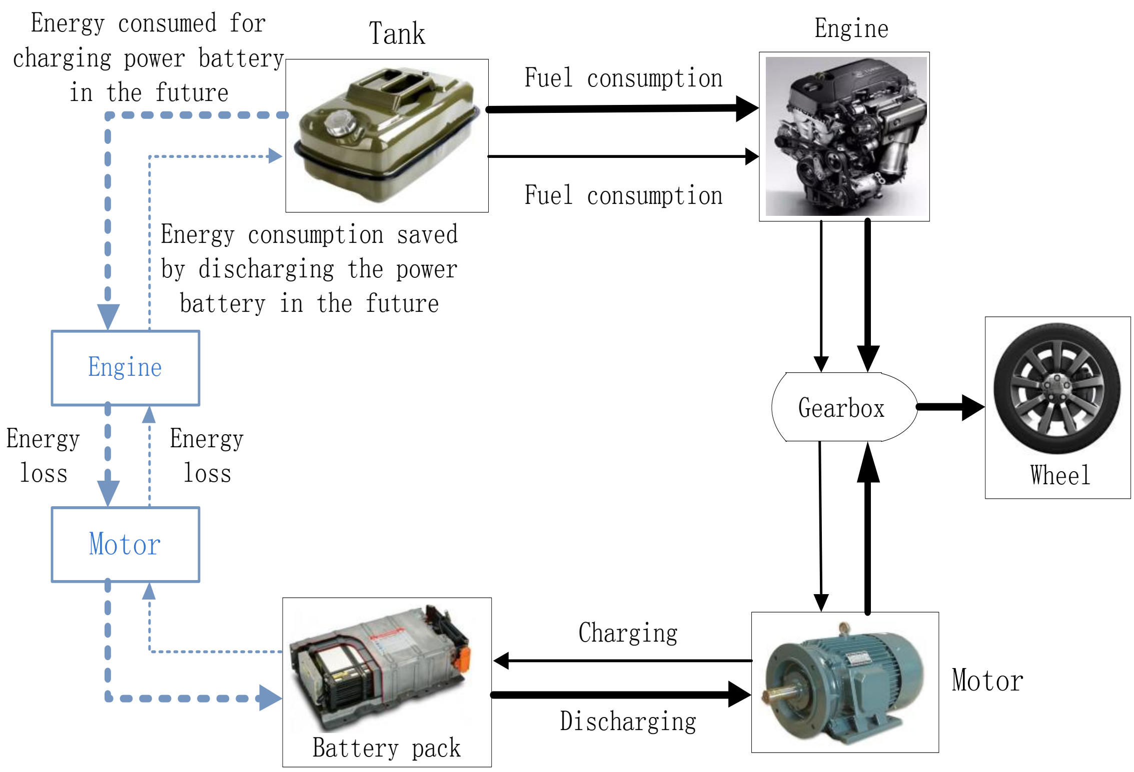

Fig. 2. Working principles of ECMS in the state of battery charging and discharging.

The equivalent fuel consumption factor determines the oil-electric conversion efficiency of the PHEV power system in the ECMS energy control strategy. If the equivalent factor becomes larger, the battery discharge is reduced, whereas the battery discharge is increased. Since the electric energy consumed by the vehicle is converted to fuel consumption by a certain coefficient, the total instantaneous equivalent fuel consumption of the PHEV at time t is:

$$\dot{x}(t)=\frac{\partial H}{\partial \lambda}=-\frac{V_{oc}(SOC)-\sqrt{V^2{}_{oc}(SOC)-4P_{batt}(T_m)\cdot R_0(SOC)}}{2\cdot R_0(SOC)\cdot Q_{batt}} \tag{22}$$

where, $\dot{m}_f(u(t),t)$ is the instantaneous fuel consumption of the engine, $\dot{m}_m(x(t),u(t),t)$ is the instantaneous power consumption of the electrical machinery which is equivalent to the amount of fuel consumed.

B. *Design of ECMS Energy Control Strategy*

On the premise of known driving conditions, the equivalent factor λ can take a constant, otherwise the SOC state of the battery needs feedback correction, so that the ECMS control strategy can be controlled in real time, and ensure that the battery SOC is maintained near the target value [34]. Based on the PMP control theory and ECMS control theory, the ECMS optimization objective function $J(t)$ of energy control is established.

$$J(t) = \min \int_0^t m_{f,eqv}(t)dt = \min \int_0^t (\dot{m}_f + \lambda \cdot \dot{m}_{em})dt \tag{23}$$

In the process of solving the optimization objective function of energy control offline, the variation range of the synergy state variable is very small, which can be regarded as a constant.

$$\dot{\lambda}(t) = 0 , \lambda(t) = \lambda(0) \tag{24}$$

If the synergetic state value is a constant, the first term of formula (3.1) is expressed as the instantaneous fuel consumption of the engine, and the second term is expressed as the instantaneous equivalent fuel consumption of the electrical machinery, the ECMS optimization function is very similar to Hamilton function. Therefore, the ECMS Hamiltonian function $H(x(t), u(t), \lambda(t), t)$ is established under the fixed equivalent factor invariant.

$$\mathrm{H(x(t), u(t), \lambda(t), t)} = \dot{m}_f(u(t),\ t) + \lambda \cdot f(x(t),\ u(t),\ t) \tag{25}$$

where, the synergy state variable λ is equivalent to the oil-electric equivalent factor, which is a constant for the course of driving condition.

Since the equivalent factor is constantly changing in the actual driving process, the instantaneous Hamilton function based on ECMS is:

$$\mathrm{H(x(t), u(t), \lambda(t), t)} = \dot{m}_f(u(t),\ t) + \lambda(t) \cdot f(x(t),\ u(t),\ t) \tag{26}$$

When the required torque is known, the optimal output torque sequence $S\dot{O}C$ of the electrical machinery is calculated by the relationship among the engine, electrical machinery, and total demand torque. The optimal output torque of engine is obtained as:

$$T_e^*(t) = T_{req} - T_{em}^*(t) \tag{27}$$

Where, $T_{req}(t), T_e^*(t), T_{em}^*(t)$ are respectively the torque of the instantaneous demand of the whole vehicle, the optimal distribution torque of the engine and the optimal distribution torque of the electrical machinery.

Since the PHEV has a large battery and the SOC changes greatly, the SOC rate of change will affect parameters of the battery. Differential equations of the SOC change cannot be considered in the actual work of the battery. Therefore, the equivalent factor needs to be adjusted accordingly with time.

According to the above analysis, the ECMS algorithm transforms the global optimization control problem of the hybrid power system into local optimization problem, and applied to the real-time optimization control problem. Therefore, the equivalent fuel consumption rate of the engine and electrical machinery output power under each combination is calculated within the working constraints of the engine and electrical machinery, and the control combination which satisfies the minimum equivalent fuel consumption rate of the vehicle is selected as the optimal control output state. Based on the architecture of the CD/CS energy control strategy, the CD/CS energy control strategy is replaced by the ECMS energy control strategy, and the other framework sections remain unchanged. The control model framework is shown in Fig. 3.

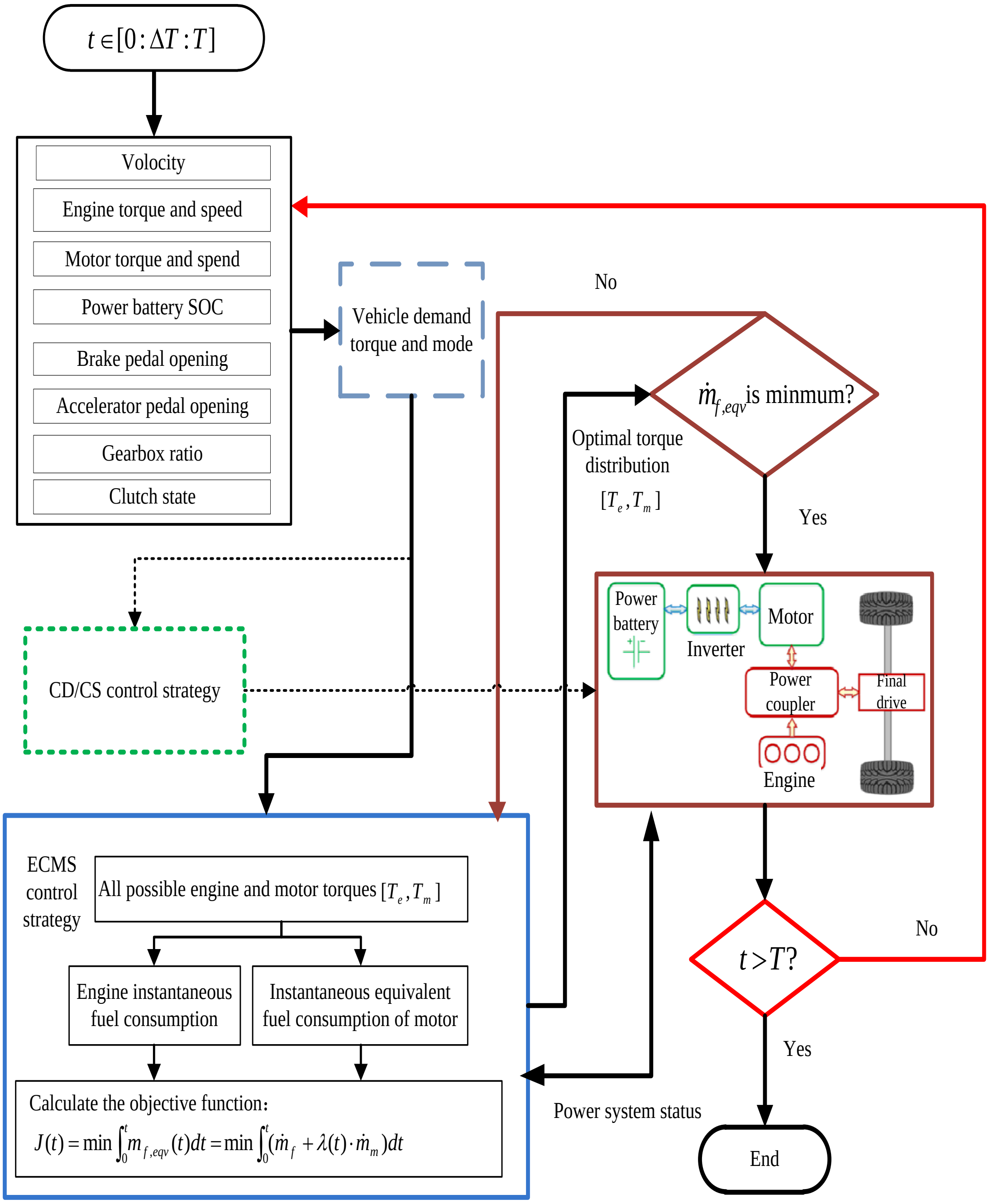

Fig. 3. ECMS energy control strategy architecture.

C. *Solving Process of the Hamiltonian Function*

ECMS objective functions are mainly solved by analog approximation. Through the analysis of the ECMS energy control strategy in the previous section, the demand torque and speed of the whole vehicle are calculated at any time, and the all output torque combinations of the engine and electrical machinery are obtained under constraints of the objective function. Then the equivalent fuel consumption rate of all combinations is calculated according to the electrical machinery efficiency characteristic diagram, engine efficiency characteristic diagram and fuel consumption MAP diagram. Finally, the optimal combination control variable is selected, which can make the engine work in the high efficiency region and minimize the objective function value. The Hamilton function solution process is shown in Fig. 4.

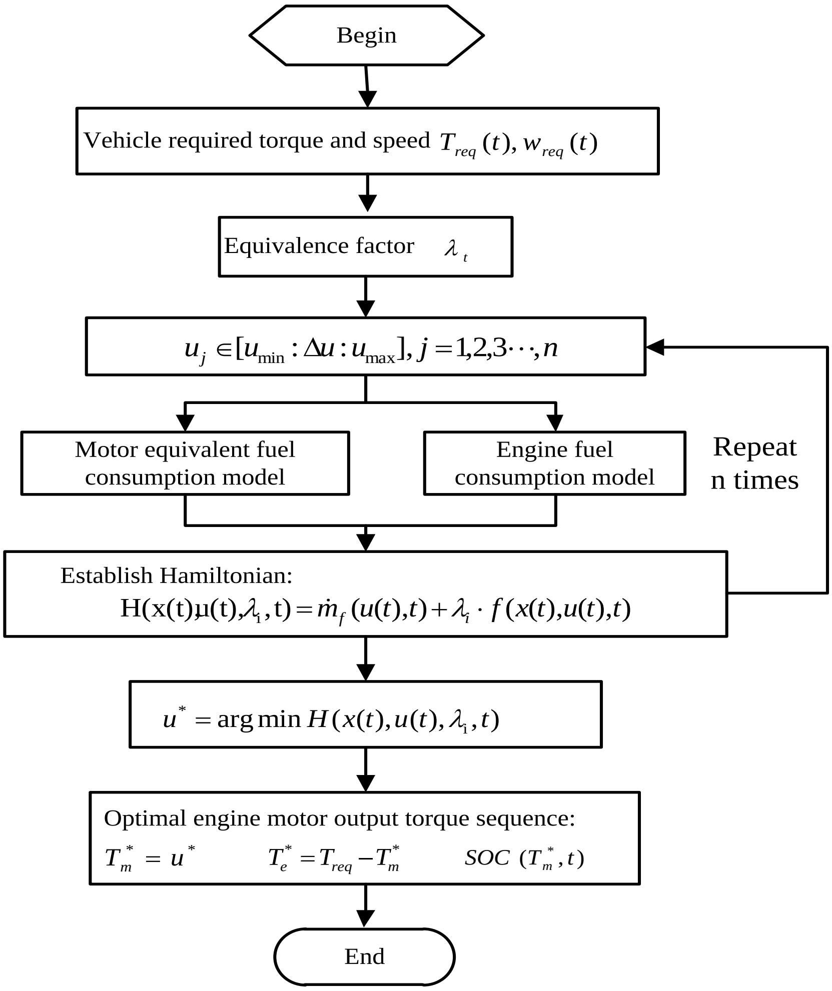

Fig. 4. Flow chart of solving Hamilton function.

The implementation steps are as follows: Determination of the control variable's range

According to parameters of the vehicle speed, battery SOC、engine output power, the total demand torque of the whole vehicle is obtained at the present time. The range of the synergistic variable is as follows.

$$T_m(t) = T_{m,n\max} : \Delta T : T_{m,p\max} \tag{28}$$

where, $T_{m,nmax}$ is the maximum generation torque of the electrical machinery, ΔT is the step length of the electrical machinery output torque variable, $T_{m,pmax}$ is the maximum drive torque of the electrical machinery.

When the electrical machinery acts as a generator, the maximum generation torque of the electric generator is as follows.

$$T_{em,\max} = \max[(T_{e,\max}(w) - T_{req}),\ T_{bat,cha\max},\ T_{em,cha\max}] \tag{29}$$

where, $T_{bat,cha\max}$ is the allowable maximum generation torque of the battery, $T_{em,cha\max}$ is the allowable maximum generation torque of the electric generator, $T_{e\max}(w_e)$ is the maximum charging torque of the engine which provide for the electric generator, $T_{req}(t)$ is the total demand torque of the vehicle.

when the electrical machinery acts as the motor, the maximum output torque of the motor is as follows.

$$T_{em,p\max} = \min[T_{req}, T_{bat,dis\max}, T_{em,dis\max}] \quad (30)$$

where, $T_{bat,dis\max}$ is the allowable maximum drive torque of the battery, $T_{em,dis\max}$ is the allowable maximum drive torque of the motor.

Through the above analysis, the output torque of the engine is as follows.

$$T_e = \begin{cases} T_{req} - T_{em} \text{ motor drive mode} \\ T_{req} + T_{em} \text{ motor power generation mode} \end{cases} \quad (31)$$

- All alternative engine operating points are determined within the current total demand torque and constraints of vehicles, and combined with engine fuel consumption map, the instantaneous engine fuel consumption $\dot{m}_e$ is obtained.
- Combined with the working efficiency MAP chart of the electrical machinery and the equivalent factor λ, the equivalent fuel consumption $\dot{m}_{batt}$ which is converted from electrical energy of the battery is calculated according to the current total demand torque of the vehicle and all the alternative motor working points.
- Repeating calculation of (2) and (3) steps, instantaneous total fuel consumption of the vehicle is calculated according to $\dot{m}_{f,eqv} = \dot{m}_e + \lambda \cdot \dot{m}_{batt}$. The control variable corresponding to the minimum instantaneous total equivalent fuel consumption $\dot{m}_{f,eqv}$ is selected as the optimal solution of the control variable at the current time. The variable is the optimal output torque T_m^* of the electrical machinery.

D. *Calculation of the Equivalent Fuel Factor*

In the process of solving the equivalent factor, the SOC value is set to 0.7, the engine and motor speed is set to 2500 r/min, the initial value of the equivalent factor is set to -3 kg, the demand torque of the vehicle is set to 90 N.m. The optimal output torque sequence of the electrical machinery is obtained by solving the minimum value of Hamilton function based on ECMS.

The maximum output torque of the engine is 150 Nm by solving Hamilton function, and the mass flow of the engine varies with the engine output torque as shown in Fig. 5.

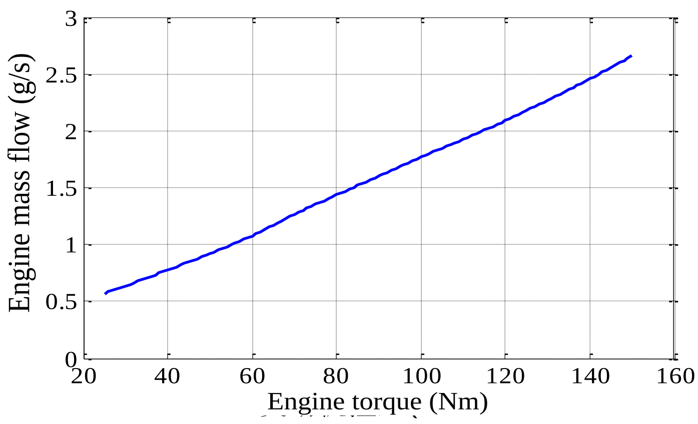

Fig. 5. The instantaneous fuel consumption of the engine varying with the output torque.

When the electrical machinery speed is 2500 r/min, the output torque of the electrical machinery is -75~75 Nm, and the instantaneous change rate of SOC with time is as shown in Fig. 6. The curve of the SOC instantaneous change rate $S\dot{O}C$ with the electrical machinery output torque is shown in Fig. 7. The change curve of equivalent instantaneous fuel consumption with output torque of the electrical machinery is shown in Fig. 7. The curve of the Hamilton function value with motor output torque is shown in Fig. 8. When the output torque of the electrical machinery is 43 Nm, the instantaneous minimum equivalent fuel consumption of the vehicle is 1.4479 g/s. Therefore, the instantaneous optimal torque distribution is the electrical machinery output torque 43 Nm and the engine torque 47 Nm.

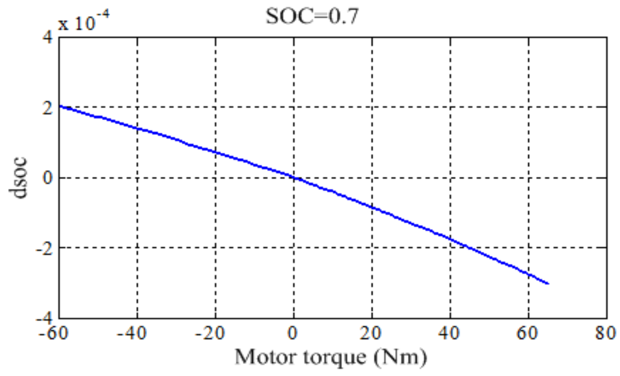

Fig. 6. The battery $S\dot{O}C$ changing with motor torque.

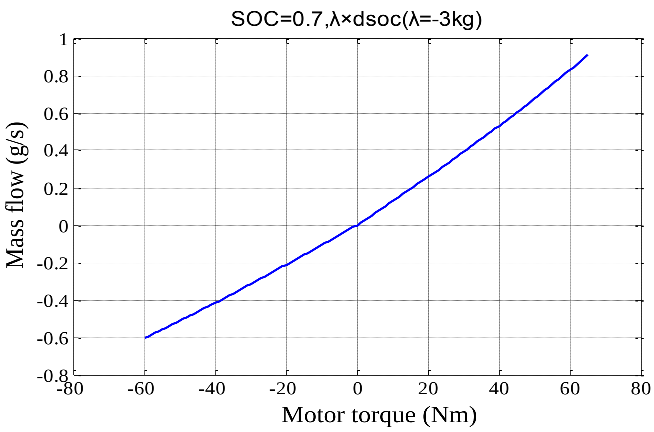

Fig. 7. Equivalent instantaneous fuel consumption of the motor output torque

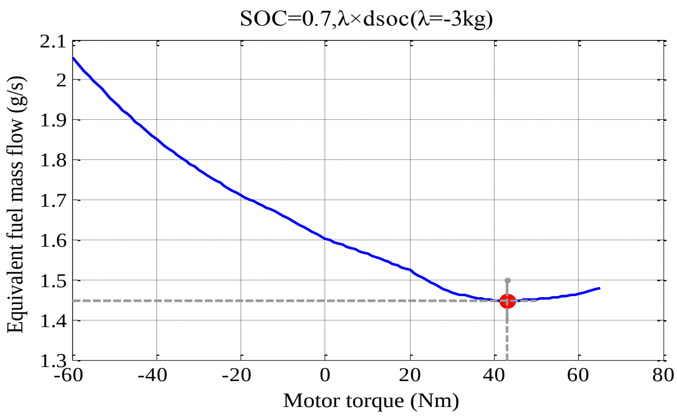

Fig. 8. Equivalent fuel consumption.

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IV. Simulation and Analysis

According to the PHEV structure, the vehicle model is built in the Cruise software and shown in Fig. 9. Based on the optimized ECMS energy control strategy control strategy, its simulink model is integrated into the whole vehicle controller by the Interface and Cruise software. Through the joint simulation, the performance evaluation of the CD/CS energy control strategy and the proposed secondary optimized ECMS energy control strategy are verified by simulation under the 6 times NEDC Driving conditions. The NEDC driving condition is shown in Fig. 10. The output torque of the engine and electric generator shown in Fig. 11 and Fig. 12 during the simulation.

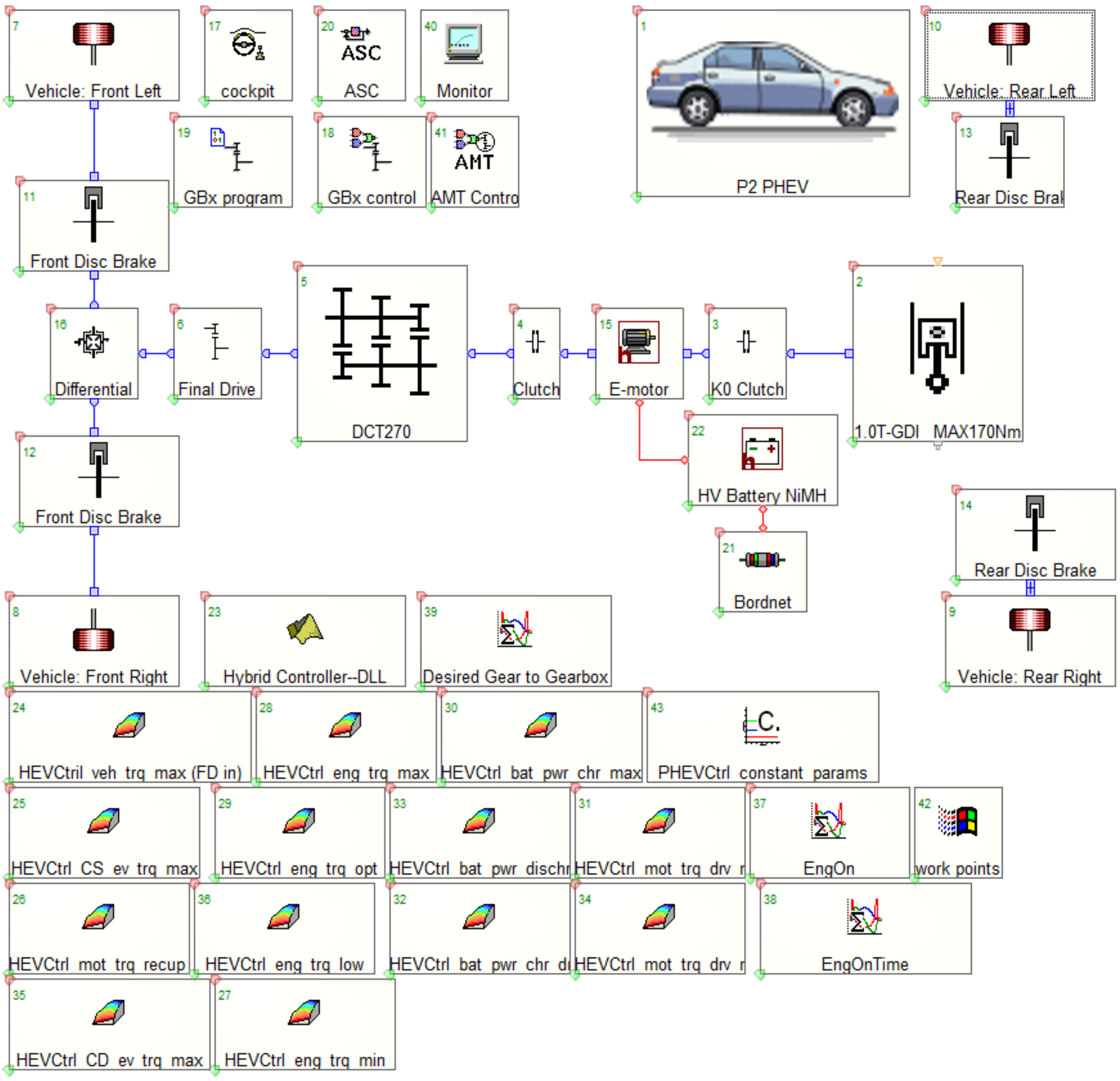

Fig. 9. Parallel PHEV model based on Cruise software.

As can be seen from Fig. 11, the CD/CS energy control strategy firstly uses the electric generator to drive the vehicle, and mainly uses the engine to work when the battery SOC reaches the lowest value. Because the engine starts frequently, the CD/CS energy control strategy increases the inefficient working range of the engine and increases the fuel consumption of the vehicle. As can be seen from the Fig. 12, compared with CD/CS energy control strategy, the ECMS energy control strategy can better make the engine and motor work, and keep the engine in an efficient working range.

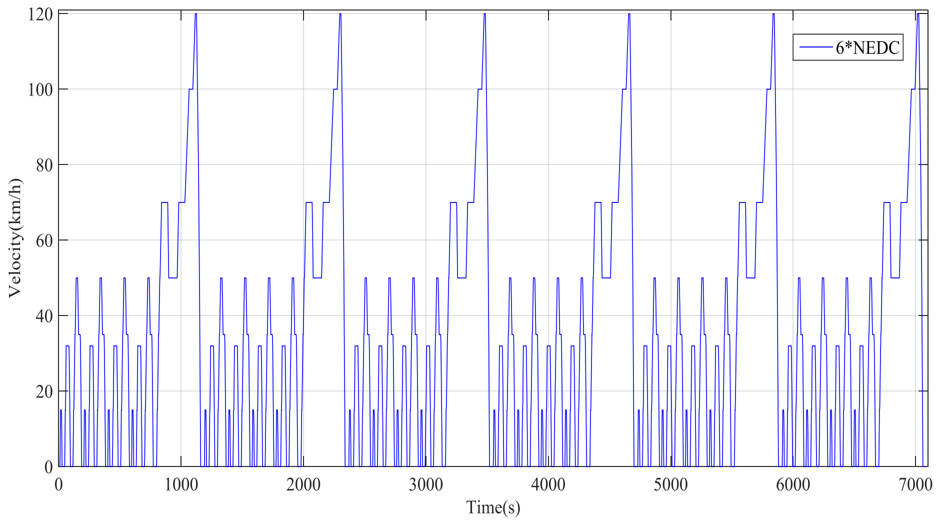

Fig. 10. 6*NNEDC cycle condition.

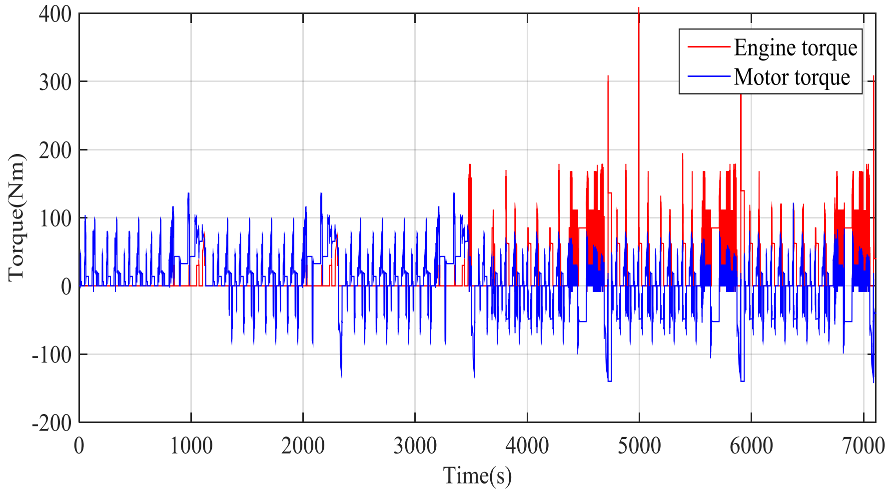

Fig. 11. Torque of engine and electric motor with time based on CD/CS control strategy.

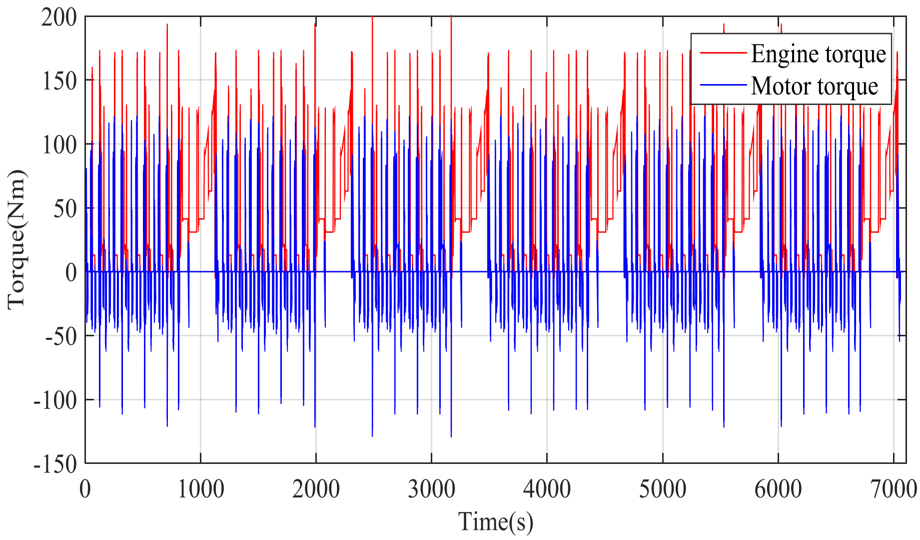

Fig. 12. Torque of engine and electric motor with time based on ECMS control strategy.

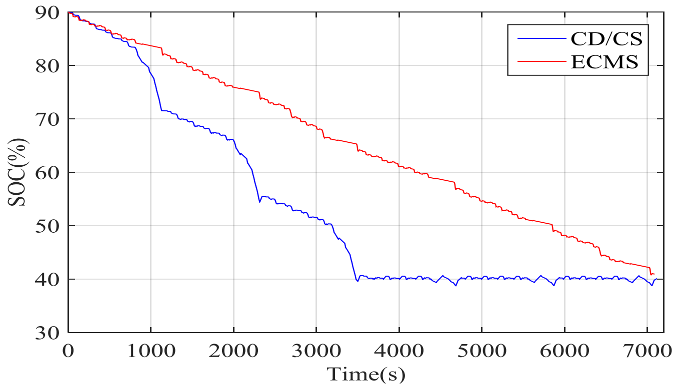

Fig. 13. Battery SOC with time.

As shown in Fig. 13, compared with CD/CS energy control strategy, the ECMS energy control strategy can make the SOC value at a certain rate, and the SOC reach the lowest value at the end made full use of the electric energy of battery.

The PHEV 100 km fuel consumption is 2.24 l/100km under the CD/CS energy control strategy. The PHEV 100 km fuel consumption is 2.04 l/100km under the optimal ECMS energy control strategy, which is 8.93% higher than the CD/CS fuel consumption.

V. Conclusions

In order to reduce the PHEV fuel consumption and improve the electricity utilization of the battery, this paper adopts the PMP and ECMS algorithm to optimize the energy control strategy of PHEV. Firstly, the parallel P2 configuration of PHEV is chosen as the object of this paper through analyzing the PHEV structures, and the PMP is selected as the optimal control algorithm by studying different algorithms of different energy control strategies. Secondly, the PHEV optimization control problem is analyzed, and the optimization control objective function of power system is established by combining the PMP theory. Considering the constraints of the control system, the objective function of the PHEV is solved. Thirdly, combining with the theory of the equivalent fuel consumption and introducing the equivalent fuel consumption factor, the secondary optimal energy control strategy of the ECMS is designed according to necessary conditions of the PMP solving the optimal problem. The process of solving equivalent fuel consumption factor is also given. Finally, based on the MATLAB/Simulink and Cruise software, the energy control strategy model of the CD/CS and optimized ECMS and the PHEV model are built and simulated by the joint simulation under the NEDC driving condition. The results show that the optimal ECMS energy control strategy can improve the CD/CS fuel economy, which ensures good performances of power and fuel economy of PHEV, improves the engine efficiency and makes the most of the battery's power.

Future studies should focus on integrating the developed procedure of the PHEV control model into a computer design support system for verifying the effectiveness of the designed control strategy by real vehicle tests [35]. Moreover, more advanced optimization algorithms need to consider the adaptive ability of the each optimal control strategy to different working conditions.

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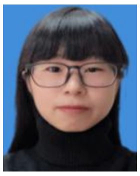

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