

# Design of an Autopilot System for Ballistic Missile

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**Abstract:** This paper aims to design an autopilot system for a ballistic missile; vanguard missile based on the lead compensation technique. The control method is used to adjust the deflection of the thrust chambers of the missile to keep it in a desired trajectory by stabilizing the system through the increase of the damping ratio of the controllers and proposing an appropriate compensation method for to eliminate the effects of any change or perturbations that may occur to the system. To design the proposed autopilot system, dynamic equations of the missile motion were derived, and the transfer functions for each element involved in the system was set and established, this includes the compensator transfer function. The system was designed and tested to assure that the resultant autopilot system satisfies the requirements of the stability. MATLAB SIMULINK blocks were used to design three different scenarios to evaluate the behavior of the system.

**Keywords:** Autopilot, Roll Stabilization, Lead Compensator, Vanguard Missile.

## 1. INTRODUCTION

A ballistic missile is a missile that follows a suborbital ballistic trajectory with the objective of delivering one or more warheads to a predetermined target. The first Ballistic missile is popularly known as V2 rocket and is developed by Germany in 1930s and 1940s under the supervision of Wernher von Braun. About 30 nations around the world have deployed operational ballistic missile. A ballistic missile trajectory consists of three parts: the powered flight portion, the free-flight portion which constitutes most of the flight time, and the re-entry phase where the missile re-enters the Earth's atmosphere. The ballistic missiles are launched in away such that the planned trajectory of these maintains the missile at zero angle of attack [1].

Unlike aircraft, there is no strongly stable position in roll for cruciform, Cartesian controlled missiles, and therefore tend due to various undesirable rolling moments such as the ones arising from airframe misalignments, a symmetrical loading of the lifting and control surfaces, Fin biases, and atmospheric disturbances. In most missiles, the roll position is required to be stabilized because the rolling motion leads to several undesirable affects [2-3].

An autopilot is a mechanical, electrical, or hydraulic system used in an aircraft, to relieve the human pilot. The original use of an autopilot was to provide pilot relief during cruise modes. Autopilots perform functions more rapidly and with greater precision than the human pilot. In addition to controlling various types of aircraft and spacecraft, autopilots are used to control ships or sea-based vehicles. The simplest form of autopilot, which is the type that first appeared in aircraft and still being used

in some of transport aircraft, is the displacement –type autopilot. It can be used to control the angular orientation of the aircraft. Designing an autopilot requires control system theory background and knowledge of stability derivatives and different altitudes for a given airplane. To achieve this it is required to determine an appropriate technique to compensate for the error. Control system compensation is the strategy used by the control system's designer to improve system dynamic performance through the addition of dynamic elements in order to mitigate some of the undesirable features of the control elements present in the system and meet the specific requirement [4-7].

Setting the gain is the first step in adjusting the system for satisfactory performance. In many practical cases, however, the adjustment of the gain alone may not provide sufficient alteration of the system behavior to meet the given specifications. As is frequently the case, increasing the gain value will improve the steady-state behavior but will result in poor stability or even instability. It is then necessary to redesign the system (by modifying the structure or by incorporating additional devices or components) to alter the overall behavior so that the system will behave as desired. Such a redesign or addition of a suitable device is called compensation. A device inserted into the system for the purpose of satisfying the specifications is called a compensator. The compensator compensates for deficit performance of the original system. Commonly used compensators and controllers are lead, lag, and lag-lead compensators and PID controllers. Also Bode diagram is always used in control system stability, When an input signal, whose magnitude varies sinusoidally with respect to time, is applied to a linear system which is stable, the output of the system will itself be a sinusoid in the steady-state, although its magnitude will be some multiple of the input signal and there may be a phase shift [8-9].

## 2. DEVELOPMENT OF LONGITUDINAL EQUATIONS OF MOTION FOR THE BALLISTIC MISSILE

To derive the longitudinal equations of the motion for the ballistic missile, it is necessary to orient the missile axis system. Fig. 1 shows an axis system for ballistic missile from which the equations of the motion are derived. The proposed missile to be discussed in this paper is the vanguard missile. Certain assumptions were made, and by applying Newton's law of motion the equations of the motion have been derived as in 1 and 2.



Equation (5) shows that the missile is unstable with poles at +1.48 and at +0.023 reminding that stability is highly required so that system maintains the best performance possible

#### 4. MODELING AND DESIGN OF THE VANGUARD MISSILE AUTOPILOT SYSTEM

The block diagram of the basic vanguard missile control system is shown in Fig. 2. Where it composed of an amplifier, compensator, servo motor and the block of the missile dynamics.

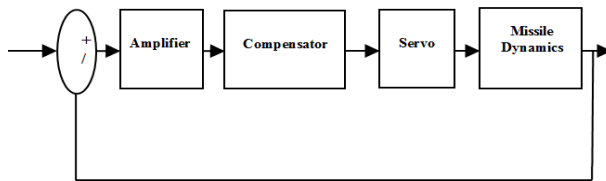


Fig.2. Block Diagram of the Vanguard Missile Autopilot System

The trajectory of ballistic-type missile in this system is planned to maintain the missile at zero angle of attack, pitch attitude must be programmed, this require certain velocity profile which may not be realized due to variation

in fuel flow rate, in addition to the presence of disturbances, any change in the angle of attack is considered to be perturbation therefore a lead compensator has been added to the forward loop to compensate for the deflections of the angle of attack. For this design, the reference pitch angle is compared with the actual attitude angle measured by a gyro to produce an error signal; this signal is amplified and fed to the lead compensator which its output activates the servo that causes the deflection of the thrust chamber.

To test the system response and characteristics, the transfer function for each element in the block diagram describing the system must be defined. The transfer function of the amplifier is described by a sensitivity gain  $K_a$ . The servo transfer function is represented as a second-order system:

$$\frac{E_a}{e} = \frac{2750}{s^2 + 42.3s + 2750}$$

As for the transfer function representing the compensator must be designed, and before that bode plot for the uncompensated system is taken to determine the design requirements. Fig. 3 is the bode diagram for the uncompensated control system

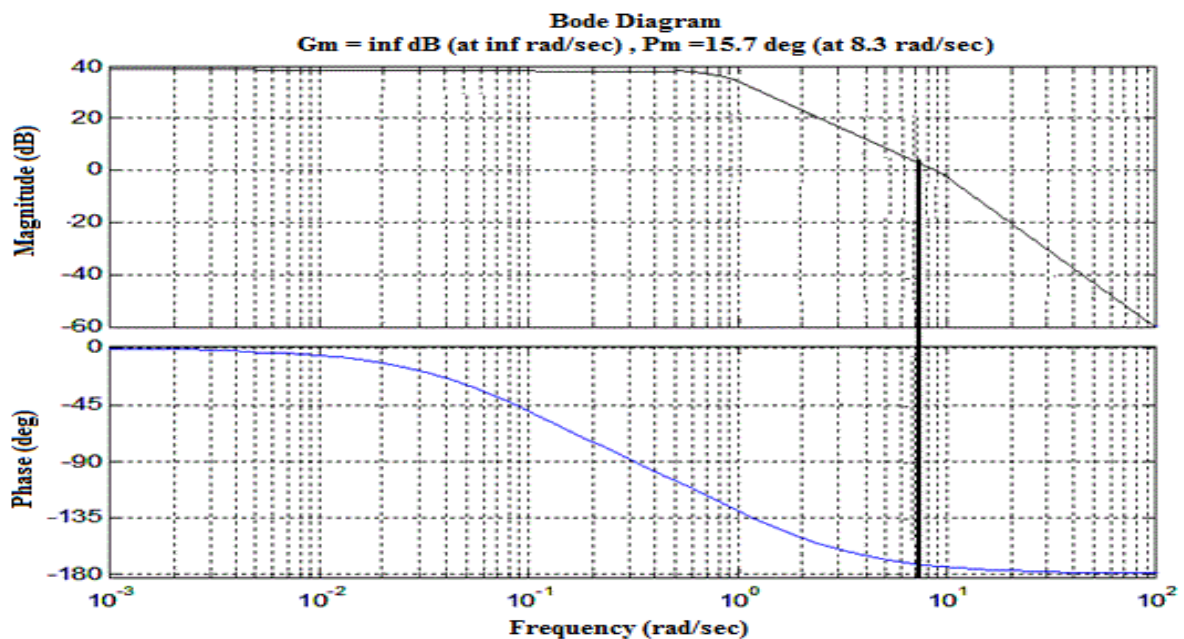


Fig.3. Bode Plot for the Uncompensated Control System

As it seen in fig (3), the phase margin is 15.7deg at frequency  $\omega_m$  equal to 8.3 radians /second. It desired to design a lead compensator with phase margin equal to 70 degree .Therefore the value to be compensated is the difference between the current phase margin and the desired phase margin.

- So a difference of  $(70 - 15.7) = 54.3$  should be compensated
- Since there is a margin of 5 degree it will become  $(54.3 + 5) = 59.3^\circ$  and substitute in the form :

$$\sin(59.3) = \frac{(\alpha-1)}{(\alpha+1)} = 0.860 \text{ From which } \alpha = 13.3$$

Therefore the compensator transfer function will be in the form:

$$T_c = \frac{1 + 13.3Ts}{1 + Ts}$$

- Now the value of T must be found.
- There is magnitude of  $T_c$  at max phase shift =  $(0.5) 20 \log \alpha$  So  $(0.5) 20 \log 13.3 = 11.238 \text{ dB}$  and

- It is know that  $\omega_m$  meets the Half of the magnitude of the max phase-lead shift. So  $(0.5)(11.238) = -5.61$  dB (-negative)
- Now from Bode  $\omega_m = 8.3$  radians/second and hence

$$T = \frac{1}{\omega_m \sqrt{\alpha}} = \frac{1}{8.3 \sqrt{13.3}} = 30$$

- And finally substituting  $T = 30$  and  $\alpha = 13.3$ , the final transfer function of the compensator is:

$$T_c = \frac{13.3 (s+2.26)}{s+30} \quad (7)$$

While the vanguard missile transfer has already been derived in equation (5), but cancelling the zero at 0.0526 with the pole at -0.023 results in the reduced order transfer function of the vanguard missile as:

$$\frac{\theta(s)}{\delta(s)} = \frac{-7.21}{(s+1.6)(s-1.48)} \quad (8)$$

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Substituting the transfer function of each element and with amplifier gain equal to 2.3 volt has resulted in the block diagram shown in Fig 4.

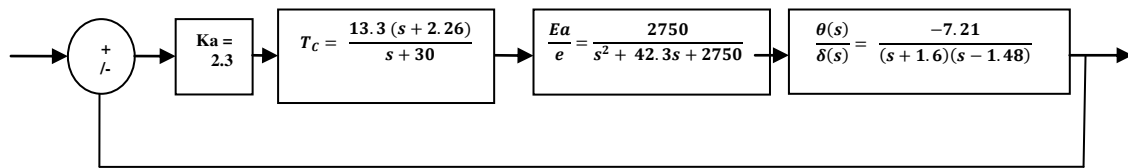


Fig.4. Vanguard Missile Autopilot System

## 5. SIMULATION RESULT AND STABILITY DISCUSSION

To make sure that the designed system meets the target response, the system was simulated and tested using SIMULINK blocks in MATLAB.

### Case 1: Uncompensated Controller

In case 1 the system has been simulated without the lead compensator, as it seen from Fig.5 the system is only composed of the servo and the missile dynamic with amplifier gain 2.3 volt. The display unit in the most top right show a negative value for step input which indicates totally instability case which is also apparently seen in Fig.6.

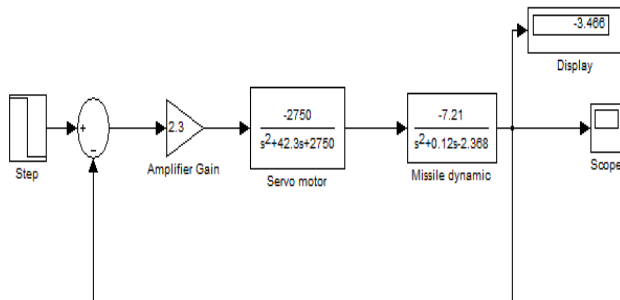


Fig.5. Simulation of the System with Uncompensated Controller

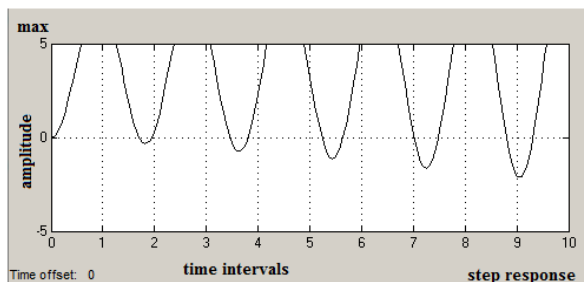


Fig.6. Step Response for the Uncompensated System Case

### Case 2: Semi-compensated Controller

In this case a lead compensator has been added to the system, the importance of the lead compensator is to stabilize the system by increasing the damping ratio. But the problem is that the value of the zero in the numerator was reduced therefore the output response for step response has negative value on the display unit. While Fig. 8 confirms that as the step response is graphically shown. There are more over shoot and the system still unstable. This case may be addressed as semi-compensated.

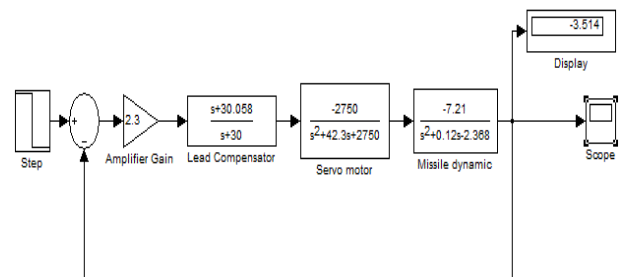


Fig.7. Simulation of the System with Semi-compensated Controller

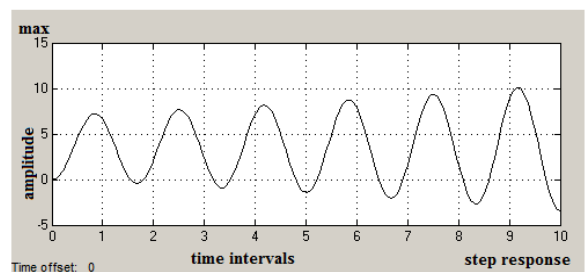


Fig.8. Step Response for the Semi-compensated System Case

### Case 3: Fully compensated Controller

In this case the lead compensator has been used with the appropriate zero (13.3s + 30.058) as it had been derived earlier. The system output for a given step response is

shown in Fig. 9 with a positive value on the display unit which indicates to stability case, meanwhile the graph in Fig.10 confirms the system is completely stable and the reason behind that is the lead compensator that keeps the missile stable and in the desired trajectory against any change or perturbations that may occurs.

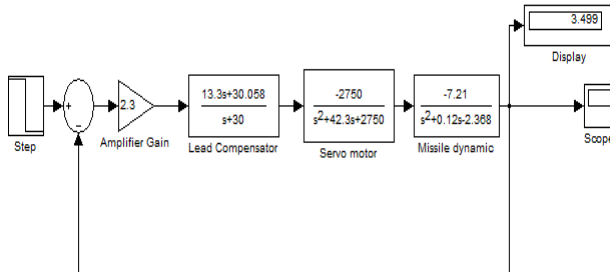


Fig.9. Simulation of the System with Completely-compensated Controller

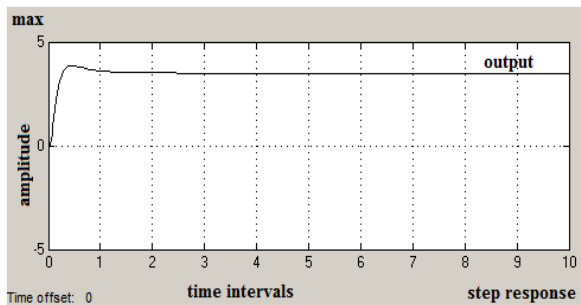


Fig.10. Step Response for Completely-compensated System Case

## 6. CONCLUSION

From the simulation results and discussion it is obvious that the missile system is stable only when the controller is completely stable, for case 1 the system was unstable and due to the absence of the compensator, while in case 2 the system has been compensated but not with appropriate values of the compensator zero and finally in case3 the system was completely compensated with the amplifier gain that equals to 2.3 volt where the system becomes greatly damped and the stability was sustained.

Therefore the vanguard missile autopilot system may be implemented in accordance to the parameters chosen for case3. It could be concluded that the resulting response shows reasonable satisfaction for the dynamic of the vanguard missile

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**Ahmed Yagoub** In 2013 Ahmed Yagoub graduated from control engineering department, faculty of engineering, AlNeelain University, Khartoum, Sudan. Since then he works as teaching assistant in the collage, he was fully involved in the electronic, control and industrial automation laboratories. In 2014 Mr Ahmed enrolled in the master program (control engineering), he successfully completed the study of the prescribed courses in the program. Curnelty he is doing his partial reseach dissertation. His reseach intreset includes control engineering and electronic engineering.



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