International Journal of Engineering Innovation & Research
Volume 4, Issue 1, ISSN: 2277 – 5668

Design of Pitch Displacement Autopilot for Business Jet Aircraft Using Root Locus

Ahmed Mohamed Yagoub Adam
Control Engineering Department, Faculty of Engineering,
ALNeelain University, Khartoum, Sudan

Mohammed Abdalla Adam Elmaleeh
Computer Engineering Department, Faculty of
Engineering, ALNeelain University, Khartoum, 12702, Sudan

Abstract: **This paper aims to design a pitch displacement autopilot for business jet aircraft based on root locus modern control system. The control method is used to determine the gains of the controller. To apply the root locus method, a model of the flight system with controllers is developed. The equation of the aircraft motion is determined and the transfer function is derived using the short period approximation. The root locus for the open loop and closed loop systems were obtained. The K values are tested for the damping ratio that keeps the system stable for any change in the gain of the controller. Finally the results of the root locus for each controller gain are analyzed and discussed. The optimum values of controller gain to achieve the desired response and keep the proposed system stable were also determined.**

Keywords: **Business Jet Aircraft, Longitudinal Autopilots, Pitch Displacement, Root Locus.**

1. INTRODUCTION

An autopilot is a mechanical, electrical, or hydraulic system used in an aircraft to relieve the human pilot. The original use of an autopilot was to provide pilot relief during cruise modes. Autopilots perform functions more rapidly and with greater precision than the human pilot. In addition to controlling various types of aircraft and spacecraft, autopilots are used to control ships or sea-based vehicles. The performance of the autopilot system directly affects the performance and mission success of the aircraft. The motion of an airplane in free flight is extremely complicated. It contains three translation motions (vertical, horizontal, and transverse) called longitudinal motions, and three rotational motions (pitch, yaw, and roll) called lateral motions and numerous structural coupled elastic motions [1]

The simplest form of autopilot, which is the type that first appeared in aircraft and still being used in some of transport aircraft, is the displacement –type autopilot. It can be used to control the angular orientation of the aircraft this autopilot was designed to hold the aircraft in straight and level flight with little or maneuvering capabilities [2]. Hence there are two types of the displacement autopilot namely pitch attitude displacement autopilot which keeps the aircraft in level flight and heading displacement autopilot which keeps aircraft in straight flight. Conceptually, the displacement autopilot works in the following manner. In a pitch attitude displacement autopilot, the pitch angle is sensed by a vertical gyro and compared with the desired pitch angle to create an error angle. The difference or error in pitch attitude is used to proportional displacement of the elevator so that the error signal is reduced. In a heading displacement autopilot, the heading angle is sensed by a directional gyro and the error signal is used to displace the rudder to reduce the error signal [3]. The pitch displacement autopilot will be discussed in this paper.

Designing an autopilot requires control system theory background and knowledge of stability derivatives and different altitudes for a given airplane. To achieve this it is required to determine an appropriate technique to compensate for the error Control system compensation is the strategy used by the control system's designer to improve system dynamic performance through the addition of dynamic elements in order to mitigate some of the undesirable features of the control elements present in the system and meet the specific requirement [[4- 5].

In this paper root locus technique will be utilized to meet the requirement of the design. The root locus plot was invented by W. R. Evans around 1948. This is somewhat surprising because the essential ideasbehind the root locus were available many years earlier. All thatis really needed is the Laplace transform, the idea that the polesof a linear time-invariant system are important in control design, and the geometry of the complex plane. Root locus is defined as a set of all points satisfying a set of conditions. The term root refers to the roots of the characteristic equation, which are the poles of the closed-loop transfer function. These poles define the time response of the system and hence the performance and stability of the system. Hence, root-locus defines a graph of the poles of the closed-loop transfer function as the system parameter, such as the gain is varied [6-7].

The basic idea behind the root-locus method is that the values of **s** that make the transfer function around the loop equal -1 must satisfy the characteristic equation of the system. The root locus is the locus of roots of the characteristic equation of the closed-loop system as a specific parameter (usually, gain K) is varied from zero to infinity, giving the method its name. Such a plot clearly shows the contributions of each open-loop pole or zero to the locations of the closed-loop poles [8].

2. LONGITUDINAL MOTION EQUATIONS AND TRANSFER FUNCTIONS

To obtain the transfer function of the aircraft it is first necessary to obtain the equations of the motion for the aircraft. The equations are derived by applying Newton's law of motion, which relate the summation of the external forces and moments to the linear angular accelerations of the system or body. To make this application, a certain assumption isneeded and an axis system should be defined. This results in considering the three longitudinal motion equations 1, 2, and 3 which represent the aircraft elevator movements [4-6].

$$(13.78s + 0.088)\acute{u}(s) - 0.392\,\acute{\alpha}(s) + 0.740\,(s) = 0 \dots\dots\dots\dots\dots\dots (1)$$

$$1.48\,\acute{u}(s) + (13.78s + 4.46)\acute{\alpha}(s) - 13.78s\theta(s) = -0.24\delta e\,(s) \dots (2)$$

$$(0.0552s + 0.619)\acute{\alpha}(s) + (0.514s^2 + 0.192s)\theta(s) = -0.710\delta e\,(s) \dots\dots\dots\dots (3)$$

Where:

α :is the angle of attack
θ :is the pitch attitude angle
δe:isthe elevator deflection

3. DERIVATION OF THE SHORT-PERIOD TRANSFER FUNCTION OF THE JET AIRCRAFT

The short-period oscillation almost occurs at a constant forward speed, thereforeú equals to zero in the equations of motion. Equation (1), which represents the motion in X direction, can be neglected, since it doesn'tcontribute much to the short period oscillation. Then the new equations of motion can be written as:

$$(13.78s + 4.46)\acute{\alpha}(s) - 13.78s\theta(s) = -0.24\delta e(s) \dots\dots\dots\dots (4)$$

$$(0.0552s + 0.619)\acute{\alpha}(s) + (0.514s^2 + 0.192s)\theta(s) = -0.710\delta e\,(s) \dots\dots\dots\dots (5)$$

And the transfer function could be evaluated as follows:

$$\frac{\theta(s)}{\delta e(S)} = \frac{\begin{vmatrix} 13.78s + 4.46 & -0.246 \\ 0.0552s + 0.619 & -0.710 \end{vmatrix}}{\begin{vmatrix} 13.78s + 4.46 & -13.78s \\ 0.0552s + 0.619 & 0.514s^2 + 0.192s \end{vmatrix}} \dots\dots\dots\dots (6)$$

The Expansion of the transfer business jet aircraft is:

$$\frac{\theta(s)}{\delta e(s)} = \frac{-1.39(s + 0.3.06)}{s(s^2 + 0.805s + 1.325)} \dots\dots\dots\dots (7)$$

4. AUTOPILOT SYSTEM MODELING AND DESIGN

The basic components of a pitch attitude control system are shown in Fig 1. The system is composed of vertical gyro, amplifier, elevator servo and the block of the aircraft dynamics.

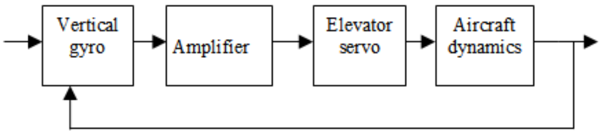

Fig.1. Pitch Displacement Autopilot General System

In this study the reference pitch angle is compared with the actual angle measured by a gyro to produce an error signal, this signal is amplified and sent to activate the elevator servo which causes the deflection of the control surface (the rudder in this case). The movement of this part forces the aircraft to achieve a new pitch orientation, which is fed back to close the loop.

To build the complete system of the aircraft the transfer function of each element in Figure1 needs to be defined. The transfer functions of the gyro and amplifier are represented by a single gain k. the elevator servo transfer function can be represented by the first-order system:

$$\frac{\delta e}{v} = \frac{Kg}{Ts + 1} \dots\dots\dots\dots\dots\dots (8)$$

Where:

δe is the deflection angle of elevator
V is input voltage
Kg is elevator servo gain
T is the servo time constant

In this research the value of the servo time constant is chosen to equal 0.1 seconds for the same aircraft model transfer shown by equation (7)

However, the problem will be encountered in determining the amplifier gain, in such a way that the control system maintains the desired performance whilekeeping in mind that the selection of the elevator servo gain (kg) must provide adequate stability margin for errors in the system

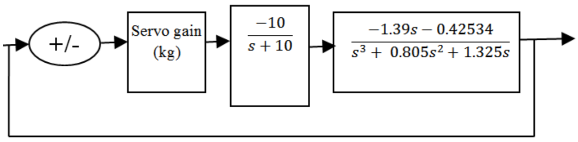

Fig.2: the autopilot System with Transfer Functions

The gain kg can be determined by taking the root locus plot for the loop transfer function. Figure 3 shows the root locus plot for the jet business pitch displacement autopilot with the damping ratio.The damping ration should be too low for achieving satisfactory dynamic response.As the gain kg increases from 0, the damping ratio decreases randomly leading to the system instability. Also for small values of kg the damping ratio of the system doesn't satisfy the requirement of the dynamic performance. The design of this system presents poor performance due to the fact that the aircraft has a very little natural frequency. TheRoot Locus for the Autopilot circuit is shown in Fig.3

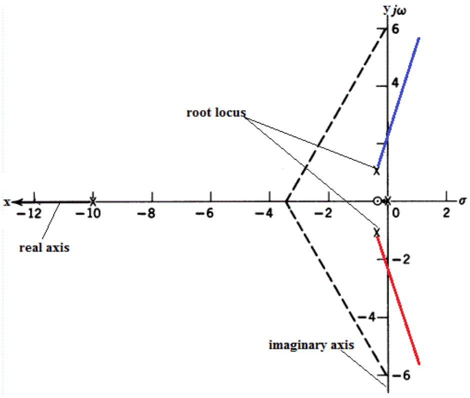

Fig.3.The Root Locus for the Autopilot Circuit

To improve the design of Autopilot system it is required to increase the damping ratio, and that could be done by adding another feedback loop. Therefore an inner feedback loop is added to the basic system shownin Fig.3.

Fig.4 represents the pitch displacement autopilot with a pitch rate feedback. In the inner loop the pitch rate is measured by a rate gyro and added to the error signal which is generated by the difference in pitch attitude [9].

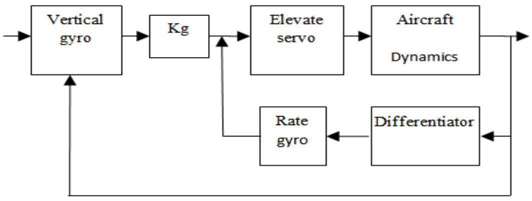

Fig.4.The Autopilot System with Two Feedback Loops

Fig.5 showsthe block diagram for the business jet aircraft autopilot when the pitch rate is added to the design. There are two feedback loops each has a gain that needs to adjust. Krg is denoted for the gain of the inner loop and kg for the outer loop. The optimum values of these gains are obtained by trial and error method.

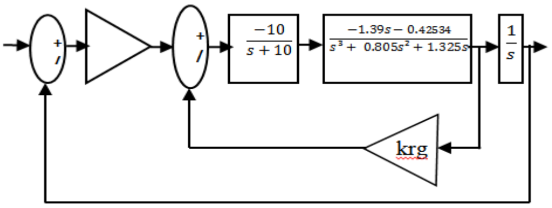

Fig.5.TheBlock Diagram of the Two Feedback Loops Autopilot System with the Transfer Functions

5. ROOT LOCUS OF THE INNER LOOP

The closed loop transfer function of the inner loop is shownin Fig. 6. It composes of the transfer functions of the elevator servo, the aircraft dynamic and the feedback gain krg respectively.

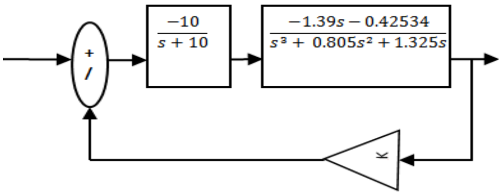

Fig.6.The Block Diagram of the Inner Loop circuit

The inner loop is assigned for the input to the elevator servo as amplifier gain and the output is connected to the aircraft dynamic. This output supposes to be compared with reference input before added to the feedback loop, but it is rather supplied to the rate gyro with Krg gain. The output of this stage is compared to the input coming from the amplifier and the difference is supplied to elevator servo. This helps adjusting the voltage supplied the servo which apparently contributes in increasing the damping ratio and hence improves the stability of the system.

Taking the root locus of the inner loop shown in Fig.7 with krg equals to 1 and 2 respectively, it that the stability of the proposed system increases keeping in mind that the closed loop poles of the inner loop become the open loop poles of the outer loop locus.

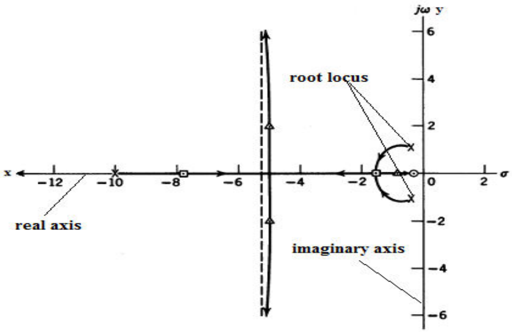

Fig.7.The Root Locus of the Inner Loop circuit

6. ROOT LOCUS FOR OUTER LOOP

The outer loop transfer function elements are similar to the previous design before including the rate gyro. However in this case the value of amplifier gain (kg),which is used to activate the servo motor; depends on the inner loop rate gyro gain (Krg). Since the gain of the elevator servo is affected by the value of the feedback inner element krg, then the outer loop locus value is dependent on the inner loop. In other words each value of the gain krg a corresponding kg must be selected and the outer loop locus is observed. This operation was iterated many times and the results have been shown in Table 1. Figure 8 represents the Root Locus for the Outer Loop of Business Jet and the Autopilot.

Table 1. The Resultant Damping Ratios for Different Values of kg and Krg

	Without Inner Loop			With Inner Loop		
	Kg=0.5	Kg = .95	Kg max =	Kg =0.5	Kg= 0.95	Kg max
Krg= 0	0.22	0.163	4.56	0.22	0.163	4.56
Krg= 1	-	-	-	0.71	0.6	14.4
Krg= 2	-	-	-	0.94	0.93	24.9

As it is seen from table1 that there is no damping ratio corresponding to the value of krg when it equals to (1, 2) and that due to the fact of the absence of the outer loop, the

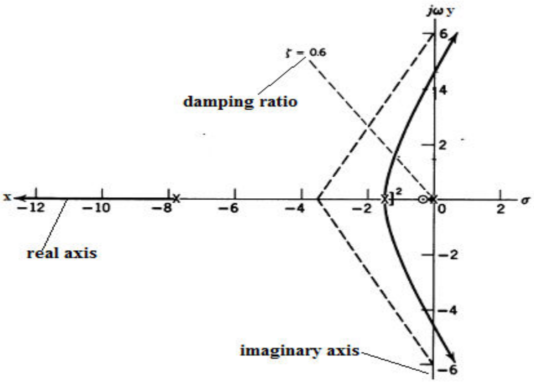

Fig.8.The Root Locus of the Outer Loop And Overall System

7. SIMULATION RESULTS

Mat Lab Version R2012 and Simulink simulation programs were used to build a code and design the for the proposed autopilot system. Fig.9 shows the final simulation of the jet business autopilot.

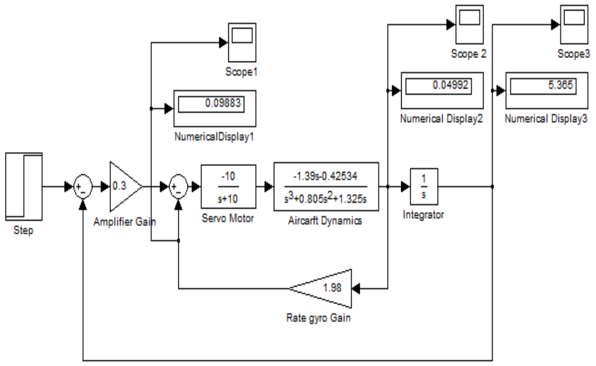

Fig.9.The Final Simulation of the Business Jet Autopilot System

As discussed in the previous sections, thissystem is composed of two loops named the inner and outer loops which primarily tend to increase the damping ration and maximize the stability. The three responses shown in Fig.9 were used to observe the output at various points in order to study the effect of the response and stability at different values of Kg and Krg.

- *Scope1*

The output of scope1 is a graphical representation of the system response for step input taken from the terminal of the rate gyro. The output is shown in Fig. 10. The graph shows some overshoot and gradually stabilizes. This indicates the affect of the rate gyro in the internal loop stability while the positive value on the numerical display1 is another indicator of system's internal stability.

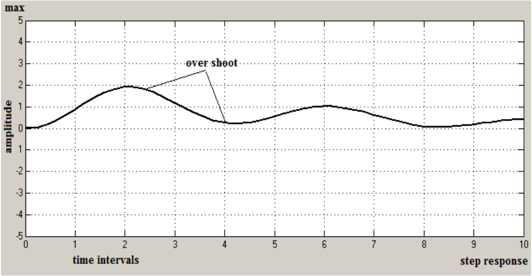

Fig.10.The Step Response of Scope1

- *Scope2*

The output of scope 2 is a graphical representation of the system response for step input taken from the terminal of the aircraft dynamic before the rate gyro. The result obtained is shown in Fig. 11. It is observed that the system has less overshoot as compared to the results shown in Figureq10.

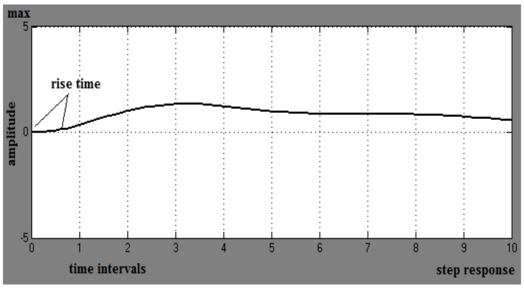

Fig.11.The Step Response of Scope2

- *Scope3*

Finally the output of sscope3 is also a graphical representation of the system response for step input and it is taken from the final output terminal as shown in Figure 12. It is observed that the graph does not include any overshoot as compared to the previous results and the system becomes stable. This indicates the effect of the outer feedback loop in the overall stability, while the positive value on the numerical display 3 is another indicator of system's overall stability

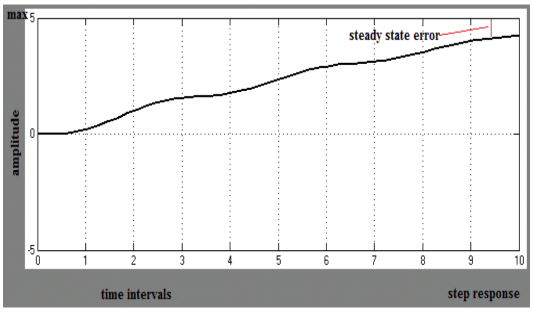

Fig.12.TheStep Response of Scope 3

8. STABILITY ANALYSIS AND DISCUSSION

From the results in Table 1 it obvious before adding the inner loop to the first system, it shows the following drawbacks: firstly the system goes unstable for any value of kg greater than zero also it is observed that there is no integration in the forward loop since the system is type0 from servo view.

By adding the inner loop feedback rate gyro to basic system it is found that the differentiator feedback inner-loop greatly enhances the system stability.

Depending on the values of the outer loop gain before and after adding the rate gyro the proposed control system is implemented to handle and compensate the pitch displacement keeping the aircraft in level flight. While the pilot must be aware of the maximum values for both Kg and Krg that hold the aircraft stable in the desired attitude before the pitch displacement autopilot is engaged.

9. CONCLUSION

From the stability analysis and discussion above and depending on the values of the outer loop gain before and after adding the rate gyro the proposed control system was implemented to handle and compensate the pitch displacement, keeping the aircraft in level flight. While the pilot must be aware of the maximum values for both Kg and Krg that hold the aircraft stable in the desired attitude before the pitch displacement autopilot is engaged.

REFERENCES

[1] R.P.G, Collin son, Introduction to Avionics, Chapman and Hall, 1996.
[2 R.C. Nelson, Flight Stability and Automatic Control, Second Edition, McGraw-Hill, 1998.
[3] John. H. Blakelock, Automatic Control of Aircraft and Missiles, John Wiley & Sons Iic, 1991.

[4] B. Stojiljkovic, et al, “The Application of the Root Locus Method for the Design of Pitch Controller of an F-104A Aircraft”, Journal of Mechanical Engineering, Vol 55, 2009.
[5] R. Langton, Stability and Control of Aircraft Systems: Introduction to Classical Feedback Control, John Wiley & Sons Ltd, 2006.
[6] William S. Levine, the Control Handbook, Ashwin J. Shah, Jaico Publishing House, 121, M.G. Road, Mumbai 400 023, 1999.
[7] Rao, V. Dukkipati, Analysis and Design of Control System Using MATLAB, New Age International (P) Ltd., 2006
[8] Katsuhiko Ogata, “Modern Control Engineering, fourth edition” Prentice-Hall, 2002.
[9] John Doyle, Bruce Francis, Allen Tannenbaum, Feedback Control Theory, Macmillan Publishing, 1990.

AUTHOR’S PROFILE

Ahmed Yagoub In 2013 Ahmed Yagoub graduated from control engineering department, faculty of engineering, AlNeelain University, Khartoum, Sudan. Since then he works as teaching assistant in the collage, he was fully involved in the electronic, control and industrial automation laboratories. In 2014 Mr Ahmed enrolled in the master program (control engineering), he succcessifuly completed the study of the prescribed courses in the program. Currnelty he is doing his partial reseach dissertation. His reseach intreset includes control engineering and electronic engineering.

Mohammed Elmaleeh received his BSc degree from University of Gezira (Sudan), Faculty of Engineering and Technology (Communication and Control Engineering). In 1998 Elmaleeh received his MSc in Electrical and Electronic Engineering, University of Khartoum, Sudan. From 1994-1998 Mr Elmaleeh worked as a researcher in Sudan Atomic Energy Commission. In 1999 he worked as automation engineer, QAPCO, Qatar. In 2009 Elmaleeh received his PhD degree in Electrical and Electronic Engineering, University Technology PETRONAS, Malaysia. Currently Elmaleeh works as Associate Professor at the faculty of Engineering, Alneelain University. Mr Elmaleeh published many papers in the filed of electronuic engineering. He supervised many post graduate and undergraduate students in their research projects. Dr Elmaleeh is assigned as a reviewer for several IEEE conferences and international journals. His research interest includes electronic engineering; industrial automation, control engingineering, fibre optics, wireless communication and computer engineering.