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State Vector Estimation Studies for ISRO'S New Launch Vehicle

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Abstract: **At any launch base, Real-time tracking and trajectory estimation play critical role during a satellite launch for flight safety as well as mission monitoring. A linear Kalman filter with a novel method of model compensation is designed and successfully implemented for Real time processing of radar data at Sriharikota Range. The verification and validation are effectively carried out using simulated data and the actual mission data of the previous launch vehicles. After validation, the data processing techniques are deployed successfully for launch operations. This paper describes the studies carried out on the above work to implement in Real Time trajectory estimation for 3rd generation launch vehicle LVM-3. Initially basic algorithm is explained with the model compensation technique. Afterwards data processing method for satellite launch vehicle is explained followed by application to nominal and off-nominal trajectories.**

Keywords: **Linear Kalman Filter, Flight Safety, Trajectory Estimation, Model Compensation Techniques**

I. INTRODUCTION

Real-time tracking and trajectory estimation play a critical role during a satellite launch catering to flight safety as well as mission monitoring. This demands design and development of radar data processing techniques ably supported by robust digital filters. In this paper the digital filter studies carried out for supporting the trajectory accuracy requirements for the ISRO's heavy lift launcher LVM-3 is presented [Fig.1]. LVM3 is the new heavy lift launch vehicle of ISRO for achieving a 4000 kg satellite launching capability to Geostationary Transfer Orbit in a cost effective manner [1]. With a lift-off mass of 640T, the 43.4m tall three-stage launch vehicle gives ISRO self-reliance in launching heavier communication satellites that weigh up to 4000 kg in GTO. It further has the potential to evolve into a human rated launch vehicle for a possible human space mission with a 10T payload capability to low earth orbit. This launch vehicle has two solid strap-on motors (S200) with 207 T propellant and a liquid booster stage (L110) with 110 T propellant loading. Upper stage is a high power cryogenic stage (C25) with 27T of propellant loading.

Fig 1.LVM-3 on launchPad

To meet the mission requirements at Sriharikota range, the space port of India, a long range tracking network consisting of 4 C-Band radars and two S-Band radars is established. State of the art digital filters and data processing techniques are deployed to process the raw tracking data of radars to arrive at optimal estimate of the state vector of the launch vehicle. This real time trajectory monitoring is highly essential for flight safety and mission management [2]. The state vector estimation studies and methodologies carried out for LVM-3 are delineated in this paper.

II. REAL TIME DATA PROCESSING TECHNIQUES

Radar measurements include systematic errors and random noise. Systematic errors are eliminated by calibration process. Random noise is eliminated by digital filtering. The digital filter provides estimated position and velocity for trajectory computation. It is clear from the above that for an accurate trajectory estimation of the launch vehicle, deployment of accurate digital filters is essential.

The well-known and widely used Kalman filter is 40 years young and it has been exploited for many applications ranging from putting man on the moon to monitoring fuel efficiency of a car to alert mechanical degradation using a personal computer. In this chapter the basic equations of Kalman filter are given with the assumptions used to obtain them. Given the dynamics, the measurement equations and other related parameters and measurements up to time T_K the filtering problem is to find the best optimal estimate in the sense linear, unbiased and minimum variance. A minimum variance unbiased estimate has the property that its error variance is less than or equal to that of any other estimate [3].

A. Kalman Filter structure

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The predicted state using the State transition matrix and previously estimated state is given as

$$X_k = \Phi_{k/(k-1)} X_{k/(k-1)} \quad (1)$$

Predicted measurement using predicted state and Observation matrix is

$$Z_{k/(k-1)} H_k X_{k/(k-1)} \quad (2)$$

Covariance of the predicted state is

$$P_{k/(k-1)} = \Phi_K P_{k-1/(k-1)} \Phi_{k/(k-1)}{}^T + Q_{k-1} \quad (3)$$

Kalman gain is computed using predicted state covariance, measurement noise matrix and Observation matrix

$$K_k = P_{k/(k-1)} H_k{}^T [H_k P_{k/(k-1)} H_T + R_k]\text{-}1 \quad (4)$$

State estimate from actual measurement is

$$X_{k/k} = X_{k/(k-1)} + K_k (Z_k - Z_{k/(k-1)}) \quad (5)$$

Covariance of the estimated state is

$$P_{k/k} = [I - K_k H_k] P_{k/(k-1)} \quad (6)$$

By knowing the initial state X_0 and its covariance P_0, state process noise Q, measurement noise R , the first four steps can be executed using the equations (1) to (4). As soon as measurements are available, last two equations (5) and (6) can be used to estimate the state and its covariance. The cycle is repeated for the next measurements to provide continuous filtered state estimate.

III. Process Noise Estimation

For launch vehicle trajectory monitoring, the estimated state has to be very accurate. In addition to state vector, the cross range and down range errors are to be minimized for decision making. The accuracy depends on data processing techniques deployed. Hence, the techniques that are developed for digital filters are of paramount importance. Initially the Real Time System with Quadrature Digital [QD] filter was in operation for all radar track data processing of rockets launching conducted from SHAR. The QD filter, used for state estimation does not provide required accuracies in state estimation and IIP computation. Hence, Kalman filter is selected[4].

A continuous effort was made to improve the accuracy of the Kalman filter. The Schmidt's Epsilon method, Aldrich Grabill method of process noise (Q) estimation and exponential weighting of measurements have been tried without any appreciable improvement in accuracy. The adhoc methods like limitation covariance(P), addition of noise and correlated initial covariances, did not provide accurate results.Finally a new method of model compensation has been developed that provides smooth and accurate trajectory and Instantaneous Impact Point [IIP] Plot[5].

A linear Kalman Filter with a novel method of model compensation is designed and successfully implemented for Real Time processing of radars data at the Range. The verification and validation are effectively carried out using simulated data and the actual mission data of the previous launch vehicles. After validation, the data processing techniques are deployed successfully for operations of the range.

This paper explains the studies carried out on the above work to implement in Real Time for Launch vehicle trajectory estimation. Initially basic algorithm is explained with the model compensation technique. Next LVM-3 simulated data processing method is explained followed by application to off nominal trajectories. Selection of different filter parameters is discussed. Launch vehicle flight data processing and editing procedure is also explained. The details of optimization of generalized Kalmanfilter for real time use is also provided.

IV. Model Error Estimation by Ideal State Processing

Polynomial model dynamics is commonly used in Kalman filter for estimating in real time the position and velocity of rockets during their thrusting phase. In reality, the rigid-body dynamics of rocket is described by three dimensional, six degree-of- freedom equations of motion.

Due to the mismatch between the actual dynamics and the model used in the filter, the filter diverges. To overcome this, various divergence control techniques varying in complexity from the use of aging factor to state augmentation are employed. The former requires extensive experimentation while the latter increases the filter order.

A new method is proposed to estimate the model uncertainty. This method uses the state obtained by solving rigid body dynamics, referred to as the ideal state, and differs from the actual state of the rocket during flight due to parameter variation, failures, etc [6],[7].

A. Model Error Estimation

In rotating geocentric frame, the acceleration of the vehicle with respect to the Earth is.

$$\bar{a} = \bar{F}/m - \bar{a}_t - 2\bar{w} \times \bar{v} \quad (7)$$

Where $\bar{a}_t$ is transport acceleration, $\bar{w}$ Earth's rotational velocity and $\bar{v}$ the vehicle velocity. The total force $\bar{F}$ consists of aerodynamic, propulsive, and gravity force and m is the instantaneous mass of the vehicle. The ideal state is obtained from (7) for the nominal vehicle parameters. The measurements generated with respect to tracking station from this state are called perfect measurements. In real time, the computation has to be carried out at a fast rate i.e., 10Hz that puts a constraint on the model. The rocket motion is approximated by a simple second-degree polynomial during the sampling interval. Since the radar measurements are in polar coordinates, slant range (R), elevation (E), and azimuth (R), the filtering is performed in this coordinate system to take the advantage of the Linear Kalman filter (LKF).

The problem is to estimate the covariance of the state noise Q_{k-1} arising due to the polynomial model in (3) instead of the rigid body dynamics in (7). By integrating (7) for the nominal vehicle parameters, ideal state X and perfect measurements Z can be computed. The perfect measurements are processed with measurement noise R = 0, as follows. From the estimated state $X_{k/k}$ predicted state $X_{k/k-1}$ is obtained using polynomial dynamics in Eq. (1). The predicted state differs from the ideal state due to mismatch in the dynamics. This difference is used as the model uncertainty in (3) for processing the next sample. Model error at (k – 1)th instant for the slant range (R) components are

$$Q_R = (R_{k/(k-1)} - R).(R_{k/(k-1)} - R)^T \quad (8)$$

$$Q_R = (\dot{R}_{k/(k-1)} - \dot{R}).(\dot{R}_{k/(k-1)} - \dot{R})^T \quad (9)$$

$$Q_R = (\ddot{R}_{k/(k-1)} - \ddot{R}).(\ddot{R}_{k/(k-1)} - \ddot{R})^T \quad (10)$$

Similar equation can be written for elevation **E** and azimuth **A**. Only diagonal terms of Q_{k-1} are estimated and off diagonal terms are assumed to be zero. Using this $Q_{(k-1)}$, $P_{k/(k-1)}$ and K_k are computed. For the next perfect measurement, state is estimated and then prediction is made. The predicted state is compared with the ideal state to estimate model error for the next sample. The above process is repeated sequentially for all the measurements throughout the flight duration. This covariance, when used for processing noisy measurements with polynomial dynamics, fully compensates the model error. The log value of a typical model compensation values for slant range is given in Fig 2. The method works because the model error is estimated by comparing state prediction by polynomial dynamics with the state obtained by integrating rigid-body dynamics that includes all the known parameters of the vehicle. Further, the estimation of Q takes into account the computational errors, since the state prediction includes them.

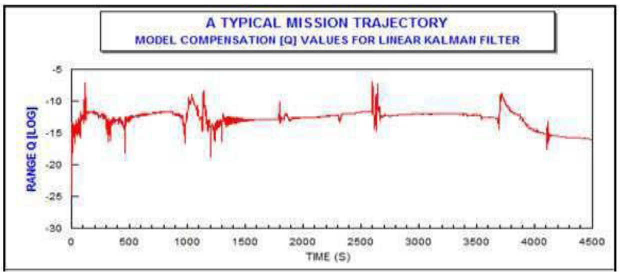

Fig 2. Log values of the model compensation for slant range

The method explained here for model error estimation differs from other methods where velocity and acceleration uncertainties are derived from the residuals obtained while processing perfect measurements. The present method which is more general and accurate, makes use of the known ideal state in estimating the uncertainties of position, velocity and acceleration.

The sequence of Q values obtained by the above procedure should be valid for the actual flight where the rocket may not follow the nominal path. In fact, the application envisaged here demands failure trajectories to be estimated as accurately as possible. For this, Q values for different failure mode trajectories were generated and found to be higher than the nominal. So a suitable weighting factor was used for the nominal Q values to take care of off-nominal conditions.

The failure modes considered are the pessimistic ones,

since large deviations from nominal are assumed so as to evaluate the filter performance. The maximum cross range errors seen for the above failure modes is of the order 50 km.

In order reduce the cross range errors, occurring for failure mode trajectories, furthers analysis is carried out for an extreme failure case. It is proper to mention here the order of magnitude of the residuals obtained for Q, both for nominal and failure trajectories. The R, E and A residuals vary for nominal from 10^{-3} to 10^{-9} depending on the occurrence of trajectory events. For the extreme failure case, for which the failure (yaw to right) occurs at 120s, residuals generated are same compared to nominal up to 120s. For failure trajectory after 120s, the range and elevation residuals remain same order as that of nominal. Due to failure causing a change in azimuth, the azimuth residuals alone increases by one order beyond 120s (10^{-6}) and by 2 orders (10^{-5}) near to 3rd stage burnout. Hence the failure mode trajectories with higher values of Q are carried out.

The value of Q studied range from 1Q to 16 Q of nominal.

$$Q_R = N \times \left(R_{k/(k-1)} - R\right).\left(R_{k/(k-1)} - R\right)^T \quad (11)$$

$$Q_{\dot{R}} = N \times \left(\dot{R}_{k/(k-1)} - \dot{R}\right).\left(\dot{R}_{k/(k-1)} - \dot{R}\right)^T \quad (12)$$

$$Q_{\ddot{R}} = N \times \left(\ddot{R}_{k/(k-1)} - \ddot{R}\right).\left(\ddot{R}_{k/(k-1)} - \ddot{R}\right)^T \quad (13)$$

where N is weight factor.

Similar equation can be written for elevation **E** and azimuth **A**[6]. The down range and cross range errors for different weight factors of process noise is given below for a typical launch vehicle.

Table.1
Down Range & Cross Range Errors with process noise (Q)

Multiplication Factor for Q	1	2	4	8	16
NOMINAL					
Down Range error (km)	6.9	7.1	7.8	8.6	10.2
Cross Range error (km)	2.8	3.0	3.1	3.2	3.4
FAILURE MODE at T+120 sec.(yaw to right)					
Down Range error (km)	7.2	7.3	7.7	8.2	9.0
Cross Range error (km)	45.3	39.0	32.6	26.4	19.1

It is candidly clear that down range and cross range errors are reasonably small for a weigh factor of 16.

B. Edit scheme for Measurements

Normally, the residuals should lie within 3 times of the covariance (3σ) of the measurements. If any of the sample lies beyond 3σ, it can be attributed either to change dynamics or larger errors in the measurements. For this mission, the edit limits for the filters are kept at 500 meters in range and 3 degrees in angles. In view of the self-editing schema available for Kalman filter, using the covariance of residuals, better and narrow edit values are chosen. Use of 3σ (3 times the standard deviation of residuals) lead to frequent editing in processing actual data. Also to take care of the large errors in the radar noise during acquisition, wider edit limits in the initial stage are preferred. Hence edit limits of 20σ for all the measurements for the first 200 samples (20sec) and later on 10σ for range and 6σ for angles is used.

C. Optimization of the Kalman filter

Generalized Kalman module with matrix notation takes around 5 milliseconds on a desk top dual core computer. This filter is optimized in the following ways without sacrificing the accuracy.

1) Independent filtering of range, elevation and azimuth facilitate decoupled filtering for each parameter, thereby matrix inversion gets reduced to a simple division in Kalman gain computation.

2) Even for each parameter matrix operations are avoided and scalar operations are used. This enabled in reducing the processing time by not accessing double and single arrays.

3) Only upper triangular elements of the covariance matrix alone are computed, since it is symmetric. The above optimization reduced the processing time to about 1 millisecond on the same computer for each cycle.

V. Results And Discussion

For the study, tracking data is generated at 10 Hz w.r.t the radars. It consists of Count Down Time [CDT] tagged slant range, azimuth, elevation, signal strength, tracking mode and process ability details. The tracking accuracies of C-Band radar are considered as 10 m in slant range and 1 millirad in angles. Process noise [Q] is generated using the established technique of model compensation by ideal state estimation. Then the track data is subjected to the LKF is for LVM-3 flight data and the analysis carried out. The Q values generated for simulation nominal trajectory is scaled and used for all off-nominal and failure mode trajectories. The state (X_o) is supplied by the P-filter by processing the first 40 samples.

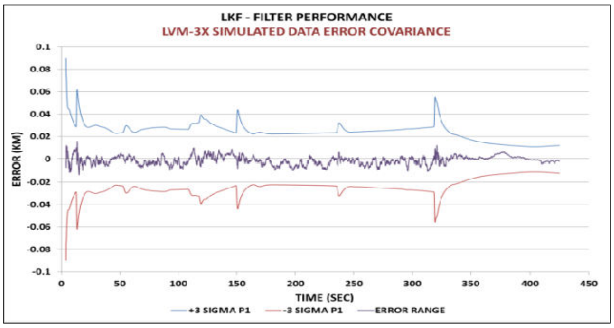

Fig 3. Range Error Covariance

The initial state covariance is same as the mentioned for the filter initialization study (1.0 Km, 0.05°,0.001°) and the measurement covariance [R] is also same as nominal

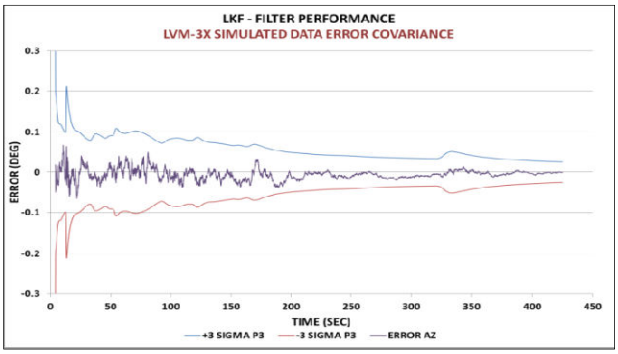

Fig 4. Elevation Error Covariance

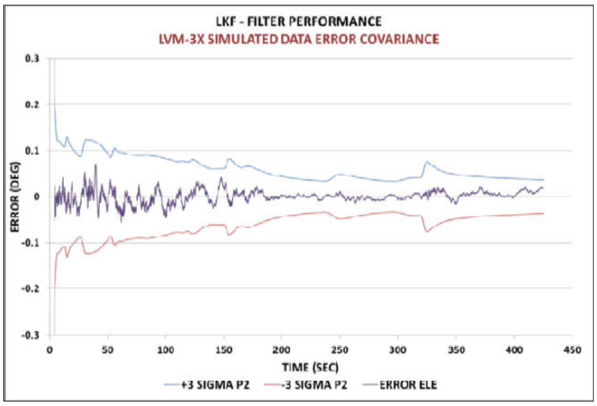

Fig 5. Azimuth Error Covariance

(10m and 0.05°).**Fig 3** to **6** provide the errors in tracking parameters as well as position error and velocity errors obtained for flight and the plots are very smooth. The errors in the tracking parameters are found to be within their theoretical bounds establishing the correctness and stability of the filter throughput the tracking period of the radars

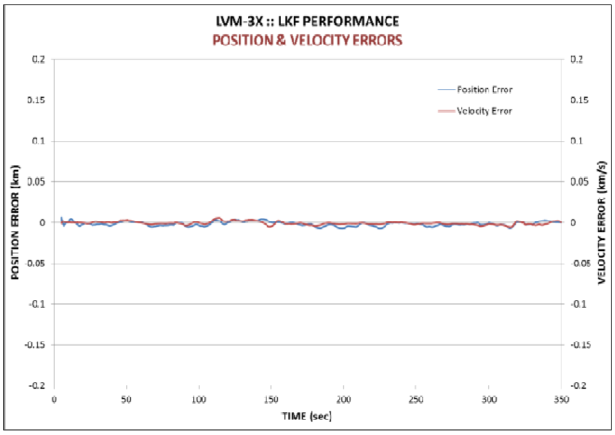

Fig 6. Position and Velocity errors

The position and velocity errors are less than 10m and 1 m/s validating the filter for trajectory monitoring and generation antenna pointing information for tracking radars to aid acquisition/ re- acquisitionof vehicle.

VI. Conclusion

After using QD filter for nearly a decade for offline purposes and real time application for launch vehicles, continuous effort has been made to evolve a simpler, fast and accurate Kalman filter for flight safety applications. A new approach to estimate model uncertainty by processing the ideal state has been established. The simulation results of nominal and failure mode trajectories as well as real flight data processing show the potential of this method for accurate real-time state estimation for flight safety application. The results presented on LKF in this paper clearly establish its versatility and same is successfully deployed for radar data processing of satellite launch vehicles including LVM-3 at SDSC SHAR.

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