International Journal of Engineering Innovation & Research
Volume 4, Issue 1, ISSN: 2277 – 5668

The Mobile Autonomous Navigation Using the Optical System

Kalimoldayev M.N., AmirgaliyevYe.N., Musabayev R.R., Nurtazin A.T.

Abstract: **Digital optical system is a normal camera internal screen which is divided into small squares of equal size (pixels), which as a result of photographing acquire averaged uniform color. At the same time, in the upper left part of the screen is highlighted the point of reference and plotted two semitransparent digital axis - left to right - *OX* and from top to bottom - *OY*. In some cases, the following question shows practical interest:**

- Can we identify the location of used digital optical system by the received image?

It is known [1], that in the case where at the picture, the images that are not lying on one line of three point objects, the assigned problem can have no more than four decisions. There's also noted that the proposed method can be applied to autonomous navigation of aerial vehicles when in the space are pre-selected and localized the individual point objects. If the flight speed of the under review vehicle is material, and its autonomous navigation must be carried out fast enough. In this situation there is actual assignment of sufficient conditions for the uniqueness of the solution. Under these conditions we propose two such natural conditions, the second of which may have practical importance.

Keywords: ***P*oint, Image, Photos, Optimal System, Triangular Pyramid, Vector, Space, Coordinate.**

I. Introduction Statement of the General Problem

One of the most important problem in the field of computer vision is the problem of calculating the camera coordinates of taken picture. The information obtained in the course of its solutions can be used for various application problems. Based on real practical situation, as well as the requirements for the calculation results, this problem can be solved with different initial conditions. For example, in [1] provides a formulation and solution of the problem of determining the coordinates of the mobile device on the space based on the images obtained by its camcorder. It is uses one camcorder which mounted on a mobile device. With camera receives a constant stream of images. Performed pair wise comparison of images and on the basis of differences between them calculated value of shear of mobile device. In this paper, as a tracking algorithm uses a modified version of the Lucas-Kanade algorithm [2]. Then, using the internal parameters of the camera, obtained during calibration [3], as well as using the rotation matrix of the camera and its shift vector, calculating the camera position change in the space based on two images. In work [4] propose the method for determining the coordinates and motion parameters of the object: course and speed. The method is based on determining the relative size of the object image on the screen size of the television camera image of the object on the screen and by lateral displacement of the image relative to the central axis of the screen. The algorithm for calculating coordinates, course and speed of the object using a fixed or rotary camera is given. To determine the orientation of the camera can also be used a priori knowledge of the visible structure of the observed objects. Thus, in [5], a new approach of calculating the orientation of the camera relative to the plane coordinate of some technical objects, as well as with respect to the global coordinate space is presented. This approach is based on the observation that man-made technical objects have ruled the visible structure, lines which extend along the three principal orthogonal directions belonging to the Cartesian coordinate system. Based on modeling the camera as orthogonal a nonlinear system of equation is obtained, the solution which allows obtaining the three angles that describe the orientation of the camera relative to the scene. To determine the projected major axis of the world coordinate system the analysis of the logarithmic magnitude spectrum of the used images by algorithm that uses information about the lines directly from the input image is performed. In work [6], a mathematical formulation of the problem of determining the position and orientation of an unmanned aerial vehicle (UAV) during a flight based on vision systems. For auxiliary purposes the possibility of additional synchronous of using of artificial Earth satellite (AES) mounted with a vision system with the same parameters considered. To solve the problem is proposed to use the reference pictures of ground surface obtained by the AES. The method of modeling the flight of the UAV, equipped with a camera is proposed. Vision technologies are widely used in the space industry. In particular, in work [7] describes a method and algorithm for determining the coordinates of points on the planet's surface using photos taken with the spacecraft mounted with a rigidly fixed camera. To solve this problem requires carrying out pre-determination of the coordinates of the spacecraft in the five points of its trajectory, which is considered to be flat. To determine the spacecrafts coordinates is used to set in it the inertial device. This method used to determine the points' coordinates on the Earth's surface and on the other planets' surface.

II. The Simple Optical System

The simple optical system (SOS) it is the device adapted for obtain a flat image of the space's part, hereinafter referred to as the stage. SOS consists of a convex lens and a flat screen placed inside the light-tight camera. Where in the lens and the screen planes are parallel, lens is a part of camera, the outside side which is directed toward the displayed scene, and the inner - towards the screen. The optical axis of the SOS is called a line passing through its center and perpendicular to the plane of the lens. Thus, the optical axis is also perpendicular to the plane of the screen. In the SOS it passes through its center. To understand the principle of the optical system only note that the rays emanating from a luminous point located in some plane (the plane of maximum sharpness) perpendicular to the optical axis of the SOS and separated from it, by some fixed distance H, in the case of passing through the lens, collected in one and the same point of the screen. Thus, the rays passing through the optical center are not refracted. Thus, to obtain image m of the space point M, sufficient to conduct line (MO). If it crosses with the screen, then the point of intersection will be its image. Since the choice of the unit of length is arbitrary, then we take it as the distance from the optical center of the screen. Now getting the image of any plane of the figure placed in a plane of maximum clarity, should be considered homothetic mapping of some rectangular part of the plane of maximum clarity on the screen. Also, it is easy to see that the coefficient of this homothetic is the number of H. Thus the image of the segment on the screen is also cut.

If there are images on the screen m_o, m_1, m_2 three points in space, then depicted point M_o, M_1, M_2 are located on the lines (m_oO),(m_1O) and (m_2O).

In this situation, the problem naturally arises:

According to images m_o, m_1, m_2 that no lie on one line vertices M_o, M_1, M_2 of given triangle $\Delta M_oM_1M_2$ find the location of the camera in space.

We note that formulated problem can have to four decisions. Obviously, the sought-for vertices should be located on the lines which can be considered edges of the infinite triangular pyramid $OM_oM_1M_2$. Thus, we obtain the following equivalent to the original, stereometric task:

Find all cross section of the infinite triangular pyramid that equal to triangle $\Delta M_oM_1M_2$.

In [] proved that the formulated problem, if known dimensions of the triangle $\Delta M_oM_1M_2$ can have up to four different solutions.

In practice, a common optical system (with a conventional screen) corresponds to the simplest camera, which from the outset was used to obtain images. The distance from the optical center of the screen rigidly fixed, and the photographed object is placed in front of the lens in the optical axis in the plane of maximum clarity. Note that any modern camera is an optical system in which the distance from the lens to the screen is not fixed, but can be easily adjusted. Thus, the variable is the distance from the optical center to the plane of maximum clarity. Moreover, most modern Japanese digital cameras focus automatically. To do this, we photographed the scene and selected a certain area, which is near the yellow spot in the center of the viewfinder. Then the digital display is automatically moved along the optical axis and its distance from the optical center is fixed as soon reached the maximum sharpness in the image of objects in the yellow spot area.

III. Digital Display Digital Optical System

Digital screen is called a rectangular photosensitive plate, broken into squares (pixels). The camera, which is used in digital screen, is called a digital optical system. It obtaining images at all points of each pixels become monotonous color and the same brightness, which have averaged parameters of these characteristics for all points of the square. It is well known that the applied method can be used to obtain images of any accuracy given in advance (field). The quality of the image increases with decreasing size pixels.

It is now well known and widely used devices via which images obtained on a digital screen, encoded (digitized) and transferred to the computer. In [*] Ashigalivev and Musabayev indicated method to produce an image of an arbitrary scene, that consisting of a small number of luminous points at arbitrary distances from the plane of the lens. Upon receipt of images such spatial figures inevitably have a situation where the image point beyond the plane of maximum clarity. Since in this case an arbitrary representative point cannot be in the line of maximum clarity, its image on the digital screen is a light spot. In [*] indicated the method which implemented by the authors of this work in a computer program that allows you to automatically find instead of getting a finite set of light spots of their geometric centers, which correspond to the location of the image source luminous points.

IV. Cartesian System on the Screen and in Space

Let the real-space fixed some digital optical system. The photographing procedure is to obtain on the screen of the flat image that situated in front of the three-dimensional spatial optical lens scene. For the mathematical treatment of the resulting flat image is commonly used flat Cartesian coordinate system ***oxy***. At the same time, bearing in mind the photographed spatial objects, entered a flat coordinate system on the screen it is desirable to connect with associated a given optical system with three-dimensional Cartesian Systems $OXYZ$. For convenience it is natural to assume that the axis OX and OY of the associated with our optical system of the spatial system $OXYZ$ is located in the plane of the lens and for the person, considering the getting by this optical system snapshot, they are parallel and inverse by direction to the axes ox and oy marked on the camera screen. In addition, the following natural requirement for spatial input system is that its OZ axis perpendicular to the screen in the direction of the scene being photographed. Usually in any rectangular image clearly fixed two mutually perpendicular directions, "left to right" and "bottom to up". Namely in the first of these directions is located a digital screen "in width" and in the second - "in height". For our purposes the center of entered Cartesian coordinate system is naturally positioned in the center of the screen. Now, as a unit segment we take a segment of the optical center of the screen. Thus in space the spatial Cartesian coordinate system $OXYZ$ is given.

If depicted on the screen of this system is the scene of a small number of point light sources, their location in the space can be taken into account by defining their coordinates in any spatial Cartesian coordinate system. Generally, any depicted scene includes spatial objects, and the procedure of the imaging is reduced to obtaining images of visible points of surfaces of these objects. As already mentioned, each image of a plane figure which is in plane the maximum sharpness is its inverted and reduced in H times copy on the digital screen. Ashigaliyev - Musabayev algorithm shows that in this case image of all points in the space located on the optical axis are coincide. It is naturally to select this image o as the beginning of a flat Cartesian coordinates on the screen plane. It is easy to see that the image m located at the plane of maximum sharpness point M on the screen located on the left of the point o on distance one. Consider that if in planar coordinate system on the screen of the optical system coordinate of axes ox and oy pass through the point o and on the direction opposite axes ox and oy. In this case, the length of the individual segments in systems oxy and $OXYZ$ are coinciding. Then, from the simple relations of similarity, it follows that the coordinates x and y of images m of the point M are related to its spatial coordinates X, Y, Z relations $X:\frac{C1}{m}/x= Y/y = Z$. The coordinates X, Y, Z of any point of space M can be found by coordinates x, y of its image, if it known the distance Z from this point to the plane of the screen: $X = xZ; Y = yZ$.

V. General Formulation of the Problem

Currently, the fields of using the digital optical systems are very broad and rapidly expanding. Images produced by such systems are typically used by external users, and control is done from single center. At the same time there are quite a number of examples when because of difficulty of communication with the control center such images must be processed at the site (within the system) for quickly identify the location of the system in space. Conventionally, we call this type of problem the procedures of the autonomous navigation. The first natural requirements to such procedures are relatively high speed of execution and given in advance the accuracy of the estimates. It is natural to assume that initially there is a fixed space (central) coordinate system $O_c X_c Y_c Z_c$, about which found and described the location and movement of a large number of three-dimensional bodies (and easily recognizable points of their surfaces). Moreover, we can assume that there is a constant stationary source of emission (in the center $O_c X_c$ of the system $O_c X_c Y_c Z_c$), which is perceived by our system and can be used for autonomous navigation. So the first question that arises in this situation can be summarized as follows:

- Is it possible by the existing coordinates of the image of the identified point objects and knowing their spatial coordinates to determine the coordinates of the optical system?

In proposed paper, there are two natural sufficient uniqueness conditions of the definition of spatial coordinates of digital optical system having a planar coordinates that not lie on one line of images of three point spatial objects, if it is known spatial coordinates of these objects themselves.

1 The two sufficient geometric conditions for existence of unique solution.

Article [**] contains purely theoretical arguments, in which all used the numerical values were assumed to be accurate. From proved theorem it should, that the task of positioning in the space of the digital optical system by the spatial coordinates of three point beacons, and the flat coordinates of their images in the photo may have no more than four decisions. (In fact, there are examples in which all cases occur). It can be seen that the results obtained with the equation of the fourth degree are usually resolved fairly quickly and accurately. Thus, these obtained methods can be successfully applied to search a hidden camera by taken photo. At the same time, the presence of extra cases can serve as a serious obstacle to the use of found algorithms for automatic autonomous control, such as aerial vehicles, equipped with the digital optical system. In view of the above, the content of the proposed work, which gives a mathematical justification of the possibility of using the inaccurate numerical values of the used coordinates, discusses ways to minimize distortion final decisions and proves two sufficient conditions for uniqueness of the solution of the problem of positioning of the optical system is actual. Note that the second of these conditions are direct practical interest and can be successfully applied for automatic mobile autonomous navigation.

Suppose that in three-dimensional space V is selected some plane P. Then in the set of all points in space, that not lies in the plane P, the ratio can be set equivalence, considering two given points A and B are adjacent to this equivalence, if the segment connecting them does not intersect the plane P. It is obvious by this relation $V\backslash P$ multitude divided into two parts, which we call open half-spaces. If to one of them attach the plane P, then the resulting multitude is called a closed half-space, or just a half-space, certain by plane P. Thus, the two closed half-space defined this plane P intersect exactly with the plane. Three-dimensional space of any three pair wise non-parallel planes is divided into eight convex parts of the closure of each of which we call infinite triangular pyramid. Thus, the infinite triangular pyramid we name the intersection of three closed half-space. The surface of the infinite triangular pyramid consists of three planar convex corners α_0, α_1, $_2$, pair wise intersecting at its edges (number of angles are in increasing order in accordance with "right hand" rule). By analogy with the concept of dihedral angle, the triangular can be called the surfaces of the triangular pyramids. Two infinite triangular pyramids (in general, any pair of space bodies) should be considered equal, if each of them can be transferred to another some isometric transformation of the space, preserving the orientation of any non-coplanar triple of vectors. Thus,

Two infinite triangular pyramids are equal if and only if their apical angle are equal and for corners the following relations were done are performed:

$$\alpha_0 + \alpha_1 + \alpha_2 < 2\pi$$
$$\alpha_0 < \alpha_1 + \alpha_2 \qquad (1)$$
$$\alpha_1 < \alpha_2 + \alpha_0$$
$$\alpha_2 < \alpha_0 + \alpha_1$$

As already mentioned in the introduction, the process of getting through this optical system which does not lie on one line images of the three points in space M_0, M_1, M_2 reduces to the construction of infinite triangular pyramid with the vertex at the optical center O of the system and edges which passing through these three points. Verge of the pyramid consists of three "apical" convex angle:

$\alpha_0 = \angle M_1 OM_2, \alpha_1 = \angle M_0 OM_2$ and $\alpha_2 = \angle M_0 OM_1$. Note that in the space through the three given points $_0$, M_1, M_2 passes exactly one plane $(M_0M_1M_2)$. In this case, the triangle $\Delta M_0 M_1 M_2$ (with the sides and interior together) we call cross section of the pyramid $OM_0M_1 M_2$ by plane $(M_0 M_1 M_2)$. In this case we say that the cross section of the infinite pyramid $OM_0M_1 M_2$ by plane $(M_0 M_1 M_2)$ is non-degenerate. The infinite triangular pyramided can be classified by the size of their vertex angles: α_0, α_1 and α_2. Firstly, if the angles $_0$, α_1 and α_2 are sharp, then the pyramid $OM_0M_1 M_2$ we call sharp. And if they are all pointless, then the pyramid $OM_0M_1 M_2$ will be called pointless.

Using entered purely geometrical concepts can be described [**] as search in this infinite triangular pyramid of all non-singular sections of equal some advance given triangle $\Delta M_0 M_1 M_2$. At the same time, non-degenerate section of this infinite triangular pyramid can be considered as the ultimate basis triangular pyramid. Now to the basic problem can be approached from the other side:

- Can we (and how many ways), this triangle $\Delta M_0 M_1 M_2$ finish to the final triangular pyramid (with the advance given apical angles α_0, α_1 and α_2), in which the original triangle will be the basis?

Approximate methods justification. As known, in practice possible only approximate measurements and calculations. Therefore, using of any mathematical methods must be accompanied by proof of their correctness. Moreover, such estimation accuracy can also play an important role in the tactical choices used in the calculation of funds. In this case, the first important remark is that the distance between used as beacons of the point objects should be commensurate with the possible tolerances the accuracy of the solution. Therefore, when working with the items on the desktop is not necessary used as beacons the planet of the solar system.

Now let show that under this principle with small inaccuracies of the initial data the process of finding the algebraic solution is sufficiently short. And the results are close enough to the real. To prove this, let us recall that the solution procedure reduces to the construction by this triangular base of the finite pyramid with the given apical angles. In this case, all the parameters of the basis and values of apical angles are approximate. Now, note that any finite triangular pyramid uniquely specifies its six edges. In our case, the sizes of all these edges are approximate. Moreover, all the tolerances on the values of apical angles directly related to the accuracy of the measurement edges. Thus, the justification for the required use of approximate methods is sufficient to show that the tops of the two pyramids, in which the length of the corresponding edges are approximately the same, and the corresponding top of the base close, also located in close proximity. It is sufficient to note that this conclusion is correct, if only slightly changes the length of one edge. Then, the overall result is also true, as any approximate change in all parameters can be no more than six steps, which consistently and significantly changing the length of the six edges. And then the base of the finite pyramid is shifted.

Having in mind first of all the applied problem of autonomous navigation, in terms of apical angles we find two simple and easily verifiable sufficient geometric conditions for the uniqueness solution of the proposed problem. In each considered cases fully describes all triangles that can be cross-sections of the pyramids from the considered classes. α_1 and α_2

Let (a_0,a_1) and (b_0,b_1) coordinates of m and n images of the points M and N on the digital screen. Then in the spatial coordinate system associated with this optical system can be assumed that the points M and N have the coordinates $(a_0,a_1 , 1)$ and $(b_0,b_1,1)$. The cosine of the angle $\angle MON$ can be calculated by the formula: $cos(\angle MON)= a_0b_0+ a_1b_1+ 1(a_0^2+ a_1^2+ 1)(b_0^2+ b_1^2+1)$

Thus,

Angle $\angle MON$ sharp $\Leftrightarrow a_0b_0+ a_1b_1 >-1$.

Angle $\angle MON$ straight $\Leftrightarrow a_0b_0+ a_1b_1 =-1$..

Angle $\angle MON$ blunt $\Leftrightarrow a_0b_0+ a_1b_1 <-1$.

Above we saw that the infinite triangular pyramid can be classified by the size of their apical angles. Now consider the following question:

What could be the cross-section of different types of pyramids?

At the same time, fixing a certain type of pyramids, we consider the class of all possible sections of the pyramid of any type. Next, we solve the question of the possible number of identical cross sections for this type of pyramids. Note that the solution to this second question is important for autonomous navigation.

Rectangular pyramid's sectional.

As we have already agreed, the infinite triangular pyramid $OM_{0,1},M_2$, in which all the apical angles are straight, called rectangular. Now consider the problem of determining the spatial Cartesian system associated with optical system, the coordinate triples of points M_0,M_1,M_2, if their images m_0,m_1,m_2 in numeric screen have coordinates $(a_0,b_0),(a_1,b_1),(a_2,b_2)$. Recall that in fact, solved problem reduces to the determination of the third spatial coordinates $Z_{0,1},Z_2$ of points M_0,M_1,M_2. It is known that the condition of rectangularity of the pyramid $OM_{0,1},M_2$ reduced to the simultaneous fulfillment of equalities:

$a_0a_1 + b_0b_1 + 1 = 0,\ a_0a_2 + b_0b_2 + 1 = 0,\ a_1a_2 + b_1b_2 + 1 = 0\ (2).$

Since in this case we known in advance triangle $\Delta M_0,M_1,M_2$, then we can measure the length n_0,n_1 and n_2 of the sides $[M_0,M_1]$,$[M_0,M_2]$ and $[M_1,M_2]$. If these length express through the coordinates $(a_0,b_0),(a_1,b_1),(a_2,b_2)$ of images m_0,m_1,m_2 points M_0,M_1,M_2 and unknown third position Z_0,Z_1,Z_2, then there will be system of three quadratic equations with three unknowns - Z_0,Z_1,Z_2: $(a_0 Z_0 – a_1 Z_1)^2 + (b_0 Z_0 – b_1 b1 Z_1)^2 + (Z_0 – Z_1)^2 = n_2^2$ $(a_1 Z_1 – a_2 Z_2)^2 + (b_1 Z_1 – b_2 Z_2)^2 + (Z_1 – Z_2)^2 = \qquad = n_0^2$ $(a_2Z_2 – a_0 Z_0)^2 + (b_2 Z_2 – b_0 Z_0)^2 + (Z_2 – Z_0)^2 = n_1^2$

Or after the algebraic conversions:

$(a_0^2 + b_0^2 + 1)Z_0^2 + (a_1^2 + b_1^2 + 1)Z_1^2 - 2(a_0a_1 + b_0b_1 + 1)Z_0Z_1 = n_2^2$

$(a_1^2 + b_1^2 + 1)Z_1^2 + (a_2^2 + b_2^2 + 1)Z_2^2 - 2(a_1a_2 + b_1b_2 + 1)Z_1Z_2 = n_0^2$

$(a_0^2 + b_0^2 + 1)Z_0^2 + (a_2^2 + b_2^2 + 1)Z_2^2 - 2(a_0a_2 + b_0b_2 + 1)Z_0Z_2 = n_1^2$

Which, in view of (1), are equivalent:

$(a_0^2 + b_0^2 + 1)Z_0^2 + (a_1^2 + b_1^2 + 1)Z_1^2 = n_2^2$

$(a_1^2 + b_1^2 + 1)Z_1^2 + (a_2^2 + b_2^2 + 1)Z_2^2 = n_0^2$

$(a_0^2 + b_0^2 + 1)Z_0^2 + (a_2^2 + b_2^2 + 1)Z_2^2 = n_1^2$

The last three equations are simple linear system with respect to the squares Z_0^2, Z_1^2, Z_2^2 and has a unique solution:

$Z_0^2 = n_1^2 + n_2^2 – n_0^2 \quad 2(a_0^2 + b_0^2 + 1);$

$Z_1^2 = n_0^2 + n_2^2 – n_1^2 \quad 2(a_1^2 + b_1^2 + 1);$

$Z_2^2 = n_1^2 + n_0^2 – n_2^2 \quad 2(a_2^2 + b_2^2 + 1)$

Note that the inequalities:

$n_1^2 + n_2^2 - n_0^2 > 0;$

$n_0^2 + n_2^2 - n_1^2 > 0;$

$n_1^2 + n_0^2 - n_2^2 > 0:$

is a necessary and sufficient condition that the original spatial triangle is sharp. In conclusion, we recall that the found values $Z_{0,1}$and Z_2 must be positive:

$Z_0 = \frac{\mathrm{n12+\ n22-\ n02}}{2}$

$Z_1 = \frac{\mathrm{n02+\ n22-\ n12}}{2};$

$Z_2 = \frac{\mathrm{n12+\ n02-\ n22}}{2}.$

Let state this fact and its application in separate statements:

VI. PROPOSITION

I. Any sharp-angled triangles and only they can be sections of the rectangular triangular pyramid.

II. Nontrivial section of the rectangular pyramid with the different planes is different.

Corollary 1 If in the optical system three points in space relative to each other reviewed at straight angles, knowing the spatial position of these points can uniquely determine the location of the optical system.

Following sufficient condition for the uniqueness of the solution of the problem is a significant generalization of the previous and has direct and wide application for autonomous navigation for fast moving optical system.

Now we want to show the uniqueness of any non-trivial section of the infinite triangular pyramid, in which at least two apical angle $_1$ and α_2 are not sharp. Let $\Delta M_0M_1M_2$ one of these non-trivial cuts and $\alpha_1 = \angle M_0OM_2$ and $\alpha_2 = \angle M_0OM_1$. According to our agreement, it can be considered the ultimate base of the finite triangular pyramid $OM_0M_1M_2$, in which the angles $\alpha_1 = \angle M_0OM_2$ and $\alpha_2 = \angle M_0OM_1$ are not sharp. It is known that, for example, in the triangle ΔM_0OM_1 side$[M_0M_1]$, lying against the obtuse angle $\angle M_0OM_2$ more of each party $[M_0O]$ and $[M_2O\ ]$: $|M_0O| < |M_0M_2|$ and$|\ M_2O\ | < |\ M_0M_2\ |$. Similarly: $|\ M_0O\ |<|\ M_0M_1\ |$ and $|\ M_1O\ |<|\ M_0M_1|$. From the inequalities $M_1O\ |<|\ M_0M_1|$ and $|\ M_2O\ | < |\ M_0M_2\ |$, it follows that in the triangles ΔM_1OM_2 and $\Delta M_1M_0M_2$ one and the same base $[M_1M_2]$; for this reason the angles at the inequalities: $\angle M_2M_1O<\angle M_2M_1M_0$ and $\angle M_1M_2O<\angle M_1M_2M_0$. But then, for angles lying against the bottom of $[M_1M_2]$, is performed: $\angle M_1M_0M_2<\angle M_1OM_2.$ Similarly, from $|M_1O|<|M_1M_2|$ and $|M_0O|<|M_0M_2|$ follows: $\angle M_0M_1O<\angle M_0M_1M_2$ and $\angle M_0M_2O<\angle M_0M_2M_1$. So $\angle M_0M_2M_1<\angle M_0OM_{1.}$ The same arguments give $\angle M_0M_1M_2 <\angle M_0OM_{2.}$ Thus, we have proved the following property:

1^0. If the finite triangular pyramid $OM_0M_2M_1$ has angles $\alpha_1 = \angle M_0OM_2$ and $\alpha_2 = \angle M_0OM_1$ are not sharp, then each of the corners $\angle M_2M_0M_1$; $\angle M_0M_1M_2$ and $\angle M_1M_2M_0$ the base of this pyramid $\Delta M_0M_1M_2$ is less its respective apical angle: $\angle M_2M_0M_1<\alpha_0$; $\angle M_0M_1M_2 <\alpha_1$ and $\angle M_1M_2 M_0 <\alpha_2$.

Now fixe the base $\Delta M_0M_1M_2$ of the finite triangular pyramid $OM_0M_1M_2$ and assume that each of the apical angles $\alpha_1 = \angle M_0OM_2$ and $\alpha_2 = \angle M_0OM_1$ are not sharp $(\geq\frac{\pi}{2})$. If the verge $\Delta M_0O\ M_1$ turn around hand $[M_0M_1]$ to align it with the base $\Delta M_0M_1M_2$, then its vertex O compatible with some point of this plane O_2. Similarly, let the point O_1 of plane $(M_0M_1M_2)$ obtained from O by turning verges $\Delta M_0O\ M_2$ around the side $[M_0M_2]$. Then the circle circumscribing the triangles ΔM_0OM_1 and ΔM_0OM_2 inside the triangle intersect at a point N, and the inequality: $\alpha_0 + \alpha_1 + \alpha_2 < \angle M1NM2 + \alpha_1 + \alpha_2 = 2\pi.$

Let d_1 and d_2 open arcs of circles circumscribed around triangles $\Delta M_0O\ M_1$ and $\Delta M_0O\ M_2$,encloses between the points M_0 and N. Now show that the set of pairs $f = \{(O_1,O_2): O_1 \in d_1\ \&\ O_2 \in d_2\ \&\&|M_1O_1|<|M_0O_2|\ \}$ is a bijective correspondence between the points of the arcs d_1 and d_2, in which, in the specifically, the point O_1 corresponding to the point O_2. For each pair of points (O_1,O_2) attaching exactly (and is effectively) one point O_0 such that on the basis of $\Delta M_0M_1M_2$ can be completed exactly one finite pyramid with lateral verges $\Delta M_0O\ M_2$; and $\Delta M_2O\ M_0$ and $\Delta M_0O_2M_1$. Thus, for reasons of continuity it follows that

2^0. On any given base $\Delta M_0M_1M_2$ exists a unique pyramid can be completed for any triple apical angles $\alpha_0,\alpha_1,\ \alpha_2$ satisfying assumptions of: $\alpha_0 + \alpha_1 + \alpha_2 < 2\pi$; $\angle M_1\ M_0M_2<\alpha_0$; $\angle M_0M_1\ M_2<\alpha_1$ and $\angle M_1M_2M_0<\alpha_2$

Because of the importance for applications summarize the two proved points as a separate theorem:

Theorem. If the infinite triangular pyramid apical angles $_1$ and α_2 are not sharp, for any of its non-trivial cut section $\Delta M_0M_1M_2$ following inequalities hold: $\angle M_2 M_0M_1 < \alpha_0$; $\angle M_0M_1 M_2 < \alpha_1$ and $\angle M_1M_2M_0 < \alpha_2$.

On the contrary, any triangle $\Delta M_0M_1M_2$, for which the condition $\angle M_2 M_0M_1 < \alpha_0$ will done is the some unique section of the infinite triangular pyramid in which apical angles α_1 and α_2 are not sharp.

The method of application of the found condition for the mobile navigation of the numerical moving optical system. As known, when planning flight of the aerial vehicles, initially selected the finite goal (or goals) of the intended flight. Then prepared a detailed map of the space, which will be the planned travel. Then select the intended route of this travel. And finally, in the course of this route is chosen chain quickly and easily identifiable objects, which we name beacons. The essence of the proposed methodology are applications of the found conditions for the mobile navigation of the numerical moving optical system lies in the fact that the chosen system of beacons placed along the route in the system: *1: 2: 1: 2* . Then during movement of the mobile optical system in its field of view is triples selected beacons, which together with the system itself as the vertex form the finite pyramid, which satisfy the conditions of the last theorem. Thus, the presence of a conventional computing system, the conditions for autonomous navigation of moving along the planned route optical system is created.

VII. CONCLUSION

Recall that in our disposal there are examples of triples points of space and their images that not lies on one line, for which there are exactly two, three and four points in space, of which can be made the photo pictures of this triples with a given images. The author also has some sufficient conditions that our main problem has no more than two solutions. Meanwhile, it appears that the search of geometric criteria for the existence of advance given number of solutions and precise description of the available picture classes of triples points of spatial objects is a very difficult mathematical problem.

In addition, still remains actual the search of other simple sufficient conditions for the uniqueness of solution of the main considering problem.

REFERENCES

[1] A.V. Kozyreva Definition of the coordinates of the mobile device in the space on the basis of images obtained from its camera // Problems of intellectualization and quality of Informatics Systems. Novosibirsk: Institute of Informatics Systems A.P. Ershov, 2006. P. 113-131.

[2] Lucas B., Kanade T. An Iterative Image Registration Technique with an Application to Stereo Vision // Proc. of 7th International Joint Conference on Artificial Intelligence (IJCAI), 1981.

[3] Zhang Z. A Flexible New Technique for Camera Calibration // Microsoft Research, One Microsoft Way, USA, Redmond - 1998.

[4] L.A. Martynov, A.V. Koriakin, K.V. Lantsov, V.V Lantsov Determination of coordinates and motion parameters of the object based on the image processing // Computer Optics, 2012, Volume 36, №2. P. 266-273.

[5] M. Lieberei, Christian Ruwwe, Bjoern Keck, Oliver Rusch, and UdoZ?lzer. Computing camera orientation relative to a world coordinate frame by detecting its projected axes // SIGGRAPH Posters, ACM, 2008, pp. 63-67.

[6] D.N. Stepanov, I.P. Tishchenko The problem of modeling the flight of unmanned aerial vehicles based on vision systems // Software Systems: Theory and Applications: electron. Sci. journal 2011. № 4 (8), p. 33-43 URL: http: psta.psiras.ru/read/psta2011, 4, 33-43, pdf.

[7] I.V. Barmine, V.P. Savinykh, V.J. Tsvetkov Remote method of determining the coordinates of points on the surface // Herald of FSUE "SPA named after S.A. Lavochkin. 03'2013 P. 26-29. URL: http://epizodsspace.no-ip.org/bibl/vestnik-lavochkina/2013/3-2013.pdf.

AUTHORS PROFILE

KalimoldayevMaksatNuradilovich -Doctor of Physical and Mathematical Sciences (2000 year), Professor, Director of Institute of Information and Computational Technologies Sciences of Ministry of Education and Science, Kazakhstan. In 1980 he graduated from the Kazakh State University named after S.M.Kirov, Faculty of "Mechanics and Applied Mathematics" majoring in "Applied mathematics". The author of more than 100 scientific works including 4 monographs and 5 textbooks for higher education institutions. Under his leadership were defended 3 doctor's and 20 candidate's theses. He developed and explored the mathematical models of the grocery market and dispersed planning production, storage, transport and distribution of petroleum products. He explored the dynamics of specific controlled economic systems in Kazakhstan.

EdilkhanAmirgaliyev - Doctor of Technical Sciences, Professor,Corr. member of NIA RK. Head of the Computer Engineering Department.He graduated from the Kazakh State University named after S.M.Kirov. The author of more than 60 scientific works.

MusabaevRustam- Senior Research.Candidate of Technical Sciences.

NurtazinAbyz - Candidate of Physical and Mathematical Sciences. Leading researcher. In 1971 he graduated from the Kazakh State University named after S.M.Kirov, Faculty of "Mechanics and Applied Mathematics" majoring in "mathematic". The author of more than 35 scientific works. Specialist in mathematical logic.