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Numerical Simulation of Underground flow in Aquifer in Akinyele Local Government Area, Oyo State, Nigeria

B.H. Ajibola[1], K.M. Odunfa[1], O.A. Adeaga[2*]
[1]Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria
[2*]Department of Mechanical Engineering, The Ibarapa Polytechnic, Eruwa, Nigeria
*Corresponding author email id: engr.adeaga@gmail.com

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Abstract **– Groundwater is one of the major sources of renewable green energies useful to man in the form of usable source (drinking and washing), Agriculture (irrigation) and energy (hydroelectricity). Water supply in Akinyele local government Oyo State of Nigeria has been a source of concern due to increase in number of wells and sunk boreholes coupled with engineering activities caused by increase in population, capital development and complex basement structure of the subsurface which has restricted the existing Aquifers in confinement fracturing and weathering. In this paper, an interactive computer simulation model for underground flow in an aquifer which is applicable for predicting flow directions and approximate depth of aquifer from the ground surface for well and borehole depth estimations has been developed using the finite difference approach modified through Taylor's series expansion combined with Darcy's law. The aquifer is modelled by imposing a finite difference scheme on the aquifer surface grids with appropriate step-size, initial and boundary conditions using a derived groundwater steady-state equation. Geophysical survey was carried out using resistivity machine to ascertain the soil and underground rock properties such as permeability, transmissivity, conductivity and borehole producing rate. The pressure heads contour map was also recorded. These actual field results were input into the algorithm for the JAVA code for calibration and validation of the simulated results. The actual field results agreed with the simulated results, hence, confirming to the calibration and validation process.**

Keywords **– Aquifer, Calibration, Pressure Heads, Validation.**

I. Introduction

Simulation model is the construction of a mathematical model to reproduce the characteristics of a physical system or process, often using a computer, in order to infer information or solve problems (ASTM, 1984). However, the aim of this project is to develop an interactive computer simulation model of underground flow in an aquifer medium using finite difference approach. Aquifer is a saturated permeable layer of the bearing rock below the ground surface capable of providing usable supply of water. The rate of movement of ground-water depends on the type of subsurface rock materials in a given area. Aquifer capable of allowing free flow of water are: sandstones, gravels, limestone or basalts while those that tends to slow down groundwater flow such as clays, shales glacial tills and silts are called Aquitard. Groundwater may appear at the surface in the form of springs or it may be tapped by wells. Groundwater is often preferable because it tends to be less contaminated by waste and organism.

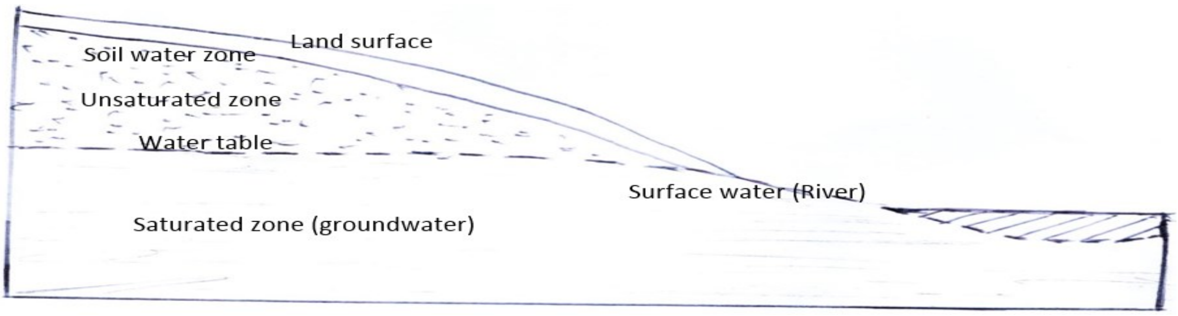

Fig. 1. Diagram showing the cross-section of an Aquifer (C.P. Kumar, NHI)

Any ground-water investigation that does more than simply collect and tabulate data involves modeling. A preliminary model describing the ground-water system is tested by collecting data. If the data fit the hypothesis, the model is accepted. Model can be:

(1) Qualitative descriptions of how process operate in a system.
(2) Simplified physical representations of the system such as sand tank and physical aquifer models and laboratory batch experiments to measure adsorption isotherms.
(3) Mathematical representations of the physical system.

Predictive Models

Simulate physical and chemical processes in the subsurface to provide estimate of how far, how fast, in what direction a contaminant may travel and depth to aquifer from top soil surface. (Heijde et al, 1985). One of the objectives of this study is to duplicate the field water levels, hence, the digital computer model is checked for its validity with the past history comparison of its response with those measured in the field. (Agbede, 1988). The groundwater

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flow equation is often derived for a small representative elemental volume where the properties of the medium are assumed to be effectively constant. The steady-state flow is described by a form of Laplace equation. (Igboekwe et al, 2011).

Geology of Akinyele Local Government

Within the local government, the dominant rock types are Banded Gneisses and Migmatites, with minor occurrences of Quartzites, Granite Gneisses, Schistose Gneisses and Augen Gneisses. The rocks are characteristic of the Nigerian Basement with the fractures trending dominantly in the North-South direction. The fractures are more intense within the Quarzites than the Gneisses, but in terms of size there are larger fractures in the Gneisses than the Quarzites. The drainage network of the area display a trellitic pattern controlled strongly by fracture trends. Rocks belonging to the Older Granite suite of igneous rocks were not encountered in this local government. (Umaru and Schoenick, 1992).

Fig. 2. Map of Akinyele Local Government

II. Hydrogeological Surveying of Study Area

Within the crystalline rock regions, two types of aquifer systems occur, these are the fractured Basement rocks and the weathered Basement rocks. These two types usually occur together in the same place, although they cannot be said to be mutually exclusive because sometimes only one form occurs at a place. The fracturing of the crystalline rocks are due mainly to earth movement (tectonism). The capacity of the crystalline rocks to store, allow movement and yield water chiefly depends on the extent, size, openness and continuity of the fractures and on the degrees to which the fractures are hydraulically connected (Umaru and Schoenick, 1992).Aquifers due to weathering of the Basement rocks are mainly in-situ decomposed rocks and aggregates. Within the weathering profile overlying the crystalline Basement, water is usually encountered in the granular pores of the overburden material, and moderately weathered coarse grained Basement rocks. The water yield in such cases depends on the nature and type of the parent rock, the amount of recharge, depth of weathering and the size and dimensions of the ground water basin or reservoir. The nature of the parent rock will determine the nature of the weathered material as clayey or non-clayey, as a general rule fields rocks produce non-clayey materials while mafic rocks produce clayey weathered products. Recharge is dependent on the variation between rainy and dry season, being more in the rainy season and less in the dry season. The depth of weathering and dimensions of the reservoir determines the volume of water that can be held in storage.

III. Methodology

Site Geophysical Investigation

We carried out the geoelectrical investigation of Akinyele site, the geoelectrical method was used because the open cast or drilling method is rather tideous and capital intensive.

Geoelectrical surveying is about the most reliable means of acquiring information about the subsurface geology. In this method, the resistivity readings on the resistivity machine is the most important reading since resistivity is a fundamental electrical property of rock materials and is closely related to the lithology. The determination of the subsurface distribution of resistivity from measurements on the surface can yield useful information on the structure or composition of buried formations. The electrical resistivity is measured by passing current into the ground through a pair of current electrodes and measuring the potential difference with another pair of potential electrodes. The Vertical Electrical Sounding (VES) technique of resistivity method is the most common technique used in determining vertical variations of foundations with depth.

The resistivity machine is strategically positioned on site and the current electrode and potential electrodes are spread out from the resistivity machine, the lengths of each pair of electrodes dictate the depth to the subsurface at each successive spread, the calculation of apparent resistivity by this method assumes that the potential electrode separation (MN) is very small compared to the current electrode separation (AB) and the potential difference derived from MN represents the electric field at the mid-point between A and B. The resistivity at each depth to the subsurface layer are read on the current electrode meter on the resistivity machine. Interpretation of resistivity data involves the conversion of the field data into resistivity curves, thickness and pressure heads or piezometric curves. This paper only uses the results of field resistivity from which the conductivities at each layer of the subsurface was calculated and the pressure heads curves or contour gotten from the field investigation.

Field Geophysical Investigation

The Vertical Electrical Sound (VES) (Resistivity dependent) tables base on field investigations.

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Table 1: VES I

Vertical Distance (L) (m)	*Resistivity (r)* (Ω)	*Conductivity* $\left(\frac{l}{r}\right)(\Omega^{-1})$
1	76	0.0132
2	88	0.0227
3	89	0.0337
4	83	0.0482
6	78	0.0769
6	75	0.0800
8	72	0.1111
12	77	0.1558
15	72	0.2083
15	83	0.1807
20	96	0.2083
30	122	0.2459
40	142	0.2817
40	138	0.2899
50	153	0.3268
60	147	0.4082
70	206	0.3398
80	224	0.3571
Total		3.3883

Total conductivity = $3.3883\,\Omega^{-1}$

Average Conductivity = $\frac{3.3883}{80} = 0.0424\Omega^{-1}$

Borehole producing at (source) = $10^{-2}s^{-1}$

Well and Borehole Formation

Overburden is about 3m, 7m and 14m respectively .Total drill depth of about 40 – 70m is hereby recommended on VES 1 to create enough reservoir.

Table 2: VES II

Vertical Distance (L) (m)	*Resistivity (r)* (Ω)	*Conductivity* $\left(\frac{l}{r}\right)(\Omega^{-1})$
1	148	0.0070
2	114	0.0175
3	73	0.0411
4	52	0.0769
6	28	0.2143
6	42	0.1429
8	45	0.1778
12	53	0.2264
15	53	0.2830
15	76	0.1974
20	82	0.2439
30	96	0.3125
40	163	0.2454
40	129	0.3101
50	134	0.3731
60	153	0.3922
70	161	0.4348
80	183	0.4372
Total		4.1335

Total conductivity = $4.1335\,\Omega^{-1}$

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Average Conductivity = $\dfrac{4.1335}{80} = 0.0517\Omega^{-1}$

Borehole producing at (source) = 10^{-2} s^{-1}

Well and Borehole Formation

Overburden is about 3m, 7m and 15m respectively

Total drill depth of about 45 – 72m is hereby recommended on VES 2 to create enough reservoir.

Simulation Scheme

Fig. 3. Map showing actual field hydraulic heads of Akinyele Local Government Area.
Source: (Federal Ministry of Land and Housing) Geology Department University of Ibadan. 1991.

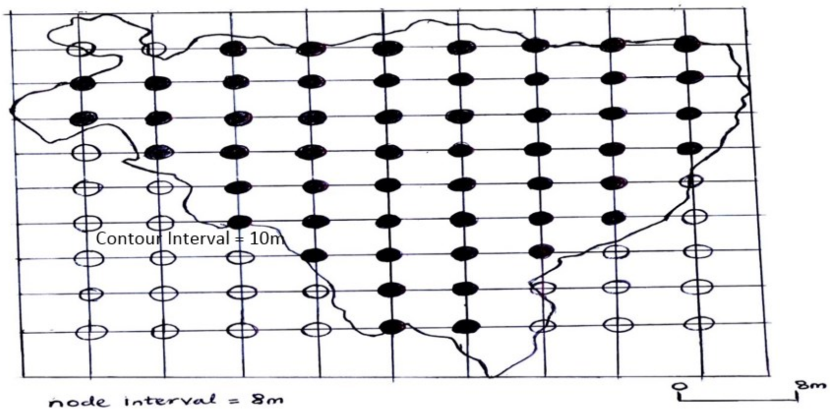

Fig. 4. Simulation Grid on Akinyele Local Government Map

The origin of the grid (that is, the upper left corner of the grid; row 1, column 1) is at a coordinate of 732'26''OE and 379'29''ON. The Akinyele domain is idealized as fractured Basement rocks and the Weathered Basement rocks aquifer. All the model cells formed by the grid for the Akinyele domain have dimension of 8m by 8m along the x-axis and y-axis. The boundaries of the Akinyele Aquifer in each case are approximated in a stepwise fashion, making some of the nodes within the model grid to be outside the aquifer areas by assigning zero transmissivity to such nodes outside the boundaries, they are excluded from the calculations.

Governing Equations

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}(K_y \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_z \frac{\partial h}{\partial z}) + R^1 = O \qquad (1)$$

Equation (1) is the governing equation for steady state heterogeneous and anisotropic flow condition for groundwater flow. For the purpose of this project we shall be looking at one –dimensional fluid flow in the y-direction. Since we are more interested in direction of flow with respect to vertical height of water column. Invoking finite difference scheme to equation (1) reduced to:

$$\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + R^1 = O \qquad (2)$$

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$$Ky\left(\frac{\partial^2 h}{\partial y^2}\right)+R^1=O \quad (3)$$

Recall from Taylor's series expansion

$$h_{j+1}=h_j+\Delta y\frac{\partial}{\partial y}h_j+\frac{(\Delta y)^2\partial^2}{2!\partial y^2}h_j+\frac{(\Delta y)^3\partial^3}{3!\partial y^3}h_j \quad (4)$$

(forward difference)

$$h_{j-1}=h_j-\Delta y\frac{\partial}{\partial y}h_j+\frac{(\Delta y)^2\partial^2}{2!\partial y^2}h_j-\frac{(\Delta y)^3\partial^3}{3!\partial y^3}h_j \quad (5)$$

(backward difference)

Adding equations (4) and (5) and neglecting degrees higher than the second order gives equation (6) to be:

$$h_{j+1}+h_{j-1}=2h_j+\frac{2(\Delta y)^2\partial^2 h_j}{2!\partial y^2} \quad ($$

By arranging, equation (6) becomes:

$$(\Delta y)^2\frac{\partial^2}{\partial y^2}h_j=h_{j+1}-2h_j+h_{j-1} \quad (7)$$

Making the derivative of the second order the subject of formula gives:

$$\frac{\partial^2}{\partial y^2}h_j=\frac{h_{j+1}-2h_j+h_{j-1}}{(\Delta y)^2} \quad (8)$$

Equation (8) is same for the x and z direction

Recall from equation (5)

$$Ky\left(\frac{\partial^2 h}{\partial y^2}\right)+R^1=O \quad (5)$$

Rearranging equation (5) gives

$$\frac{\partial^2}{\partial y^2}h_j=-\frac{R^1}{Ky} \quad (9)$$

Substituting $\frac{\partial^2}{\partial y^2}h_j$ for it finite difference equivalence from equation (9)

$$\left[\frac{h_{j+1}-2h_j+h_{j-1}}{(\Delta y)^2}=\frac{-R^1}{Ky}\right] \quad (10)$$

Equation (10) is the model algorithm

An alternative algorithm is:

Recall Darcy's law: $q=\frac{-K_y\partial^2 h_j}{\partial y^2}$ …. (11)

Substituting $\frac{\partial^2}{\partial y^2}$ for it Taylor's series finite difference equivalent gives

$$q=-K\left[\frac{h_{j+1}-2h_j+h_{j-1}}{(\Delta y)^2}\right] \quad (12)$$

$$h_j=\frac{h_{j+1}+h_{j-1}}{2}+\frac{q_i(\Delta y)^2}{2k} \quad (13)$$

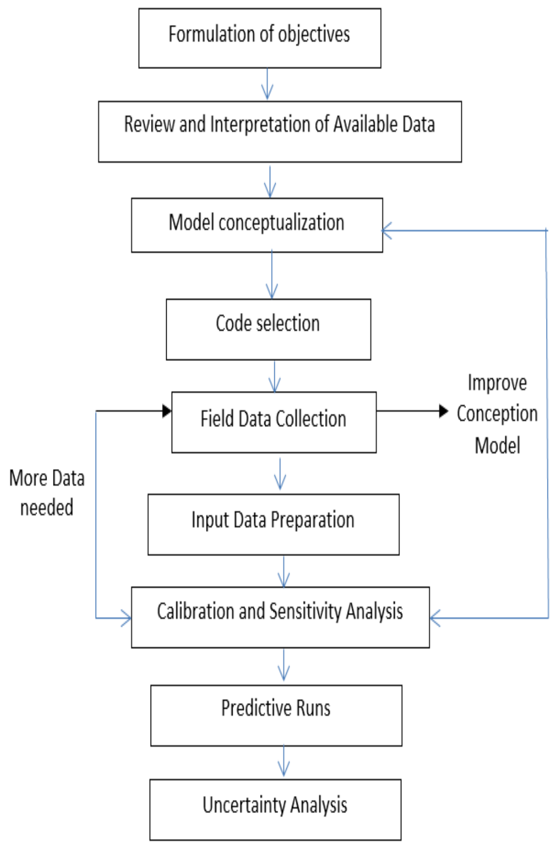
Making h_j (piezometric head in y-direction) the subject of formula from equation (12) gives equation (13):

The following field values from geological survey was input into the algorithm of the JAVA code .

Average Conductivity of subsurface soil and rock = 0.0424Ω-1

Borehole producing at (Source) = 10-2s-1.

Boundary Condition: Minimum and maximum values of hydraulic heads from the Piezometric map: 159m and 219m.



Modeling protocol

Fig. 5. Flow chart for the development of the numerical groundwater model

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IV. Results and Discussion

Table 3. Shows the result of the simulated hydraulic heads

Simulated hydraulic heads (h) in (m)	*Simulated hydraulic head range in (m)*
159	*165*
165	*171*
171	*177*
177	*183*
183	*189* Simulated values
189	*195*
195	*201*
201	*207*
207	*213*
213	
219	

To determine approximate depth to aquifer for well and borehole for the given location, the simulated range of hydraulic head i.e. (pressure heads) must be subtracted from the elevation of the location from the sea-level gotten the topographic map as shown in the diagram below.

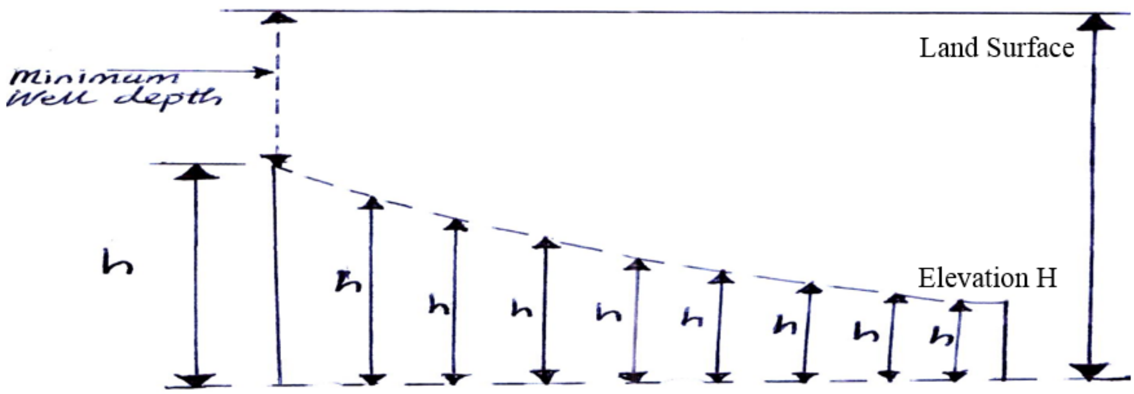

Fig. 6. Diagram showing hydraulic heads and elevation from the datum (sea level)

Table 4: shows the difference of the simulated ranges of pressure heads from the elevation of Akinyele local government area from the sea-level.

Simulate range of pressure heads in (m)	*Evaluated pressure heads from elevation in (m)*	*Depth to aquifer*
213	*229 – 213 = 16*	*16*
207	*229 – 207 = 20*	*20*
201	*229 – 201 = 28*	*28*
195	*229 – 195 = 34*	*34*
189	*229 – 189 = 40*	*40*
183	*229 – 183 = 46*	*46*
177	*229 – 177 = 52*	*52*
171	*229 – 171 = 58*	*58*
165	*229 – 165 = 64*	*64*

From the results, it can be seen that the simulated range falls between actual field values for minimum depth recommended for well and borehole. The model was assumed to be true representative of the groundwater system. After the successful calibration for the steady–state, it was noted that with average conductivity of the soil and rock properties taken from two different locations with values of $0.0424\Omega^{-1}$ for Vertical Electrical Sounding (VES 2) and $0.0517\Omega^{-1}$ Vertical Electrical Sounding for (VES 2) borehole producing at 10^{-2} s^{-1} (source) with the same hydraulic heads boundary conditions the saturated results for the two sets of reading agrees with the actual field data. When the maximum and minimum values of the simulated hydraulic heads were subtracted from elevation of Akinyele from the sea level i.e. (229 – 213) m = 16m and (229 – 165) =64m it agrees with the actual field recommendations of 15m in for well and 60-70 for borehole.

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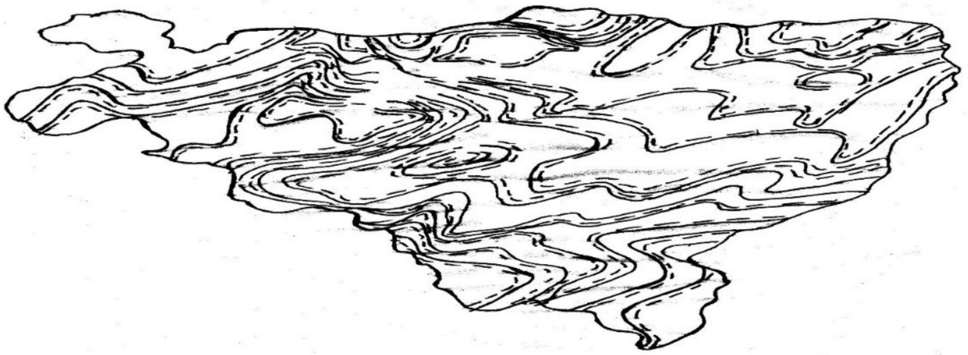

Fig. 7. The map below shows the pictorial comparism between actual field hydraulic heads and simulated hydraulic heads.

V. Conclusion

The interactive computer simulation model for the underground flow at the aquifer medium was developed, calibrated and validated, hence can be applied to any other location.

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Authors' Profiles

Engr. Ajibola Babatunde .H was born in Ibadan, Oyo satae Nigeria in 1982. He obtained Master of Science in Mechanical Engineering from the University of Ibadan, Oyo State Nigeria. In 2015.He also obtained Bachelor of Engineering in Metallurgical and Materials Engineering from the Federal University of Technology Akure, Ondo State, Nigeria.in 2006. He excellently passed West African Senior Secondary Examination from Army Comprehensive High School Akure, Owena Barracks, Ondo State 1997 First School Leaving Certificate, Army Children School II Akure, Owena Barracks Ondo State 1991He is a Member of Nigerian Society of Engineers.

Dr. K. Moradeyo Odunfa is a Senior lecturer in the department of Mechanical Engineering at the University of Ibadan, Ibadan, Oyo State, Nigeria. He lectures in the following Mechanical Engineering courses such as Applied Thermodynamics, Heat Transfer, Fluid Mechanics coupled with Energy Technology and Management. Moradeyo Odunfa conducts research on modeling of a thin-liquid falling-film in Absorption cooling systems, focusing on Magnetic field enhancement in both water-lithium bromide and water-lithium chloride absorption refrigeration systems. His interests include modeling, cooling system, energy (Renewable and Non-renewable) and environment coupled with building services engineering. He earned B. Sc, M. Phil and Ph. D degrees in Mechanical Engineering with Thermo-fluids option at the doctoral level from the University of Ibadan, Ibadan. Dr. Odunfa Moradeyo is a seasoned and practicing COREN registered Engineer who has worked in several engineering establishments notably among them are Victory Consult (Consultants in Mechanical, Electrical, Communication and Solar Energy Engineering), Olo Engineering Nig. Limited and Osot Associates, all in Ibadan. He was also a lecturer on sabbatical in thermo-fluids courses with the Covenant University, Canaan land Ota, Ogun State, Nigeria between 2013 and 2014.

Dr. Oyetunde Adeoye ADEAGA obtained B.Eng. (Hons.) Mechanical Engineering from the Federal University of Technology Akure in 1999. He latter proceeded to University of Ibadan where he obtained M.Sc. and Ph.D. degree, Mechanical Engineering in 2005 and 2015 respectively. He is currently a senior lecturer at The Ibarapa Polytechnic, Eruwa. He is a seasoned lecturer and a computer programmer. He specializes in Thermofluids (Reservoir simulation, single and multiphase fluid flow in subsurface porous media etc.). He is also a consultant to many engineering firms and organizations. He is a registered Engineer with COREN. He is member of many national and international professional societies and bodies like American Society of Thermal and Fluid Engineers, International Association of Engineers, to mention a few.

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