Nonlinear Deformation of Flexible Orthotropic Shells of Variable Thickness in The Non Stationary Magnetic Field

Indiaminov Ravshnan  
E-mail: r_indiaminov@mail.ru

Akbaev Umidd  
E-mail: revoch@bk.ru

Dustmurodov Abdumannon  
E-mail: abdumannondustmurodov@mail.ru

Abstract – The analysis of the stress state of the flexible orthotropic shells under the action of variable time and variable mechanical force on the external electric time on, based on mechanical and electromagnetic orthotropy. We investigate the effect of the thickness on the stress-strain state of orthotropic shell. Nonlinear deformation of current-carrying orthotropic conic shells in the non stationary magnetic field is studied in ax symmetric statement. The results are indicative of the effect of shell thickness on the deformation and the need to take account of this factor in the calculation scheme.

Keywords – Orthotropic Shell, Magnetic Field, Magneto Elasticity.

I. INTRODUCTION

Increased interest in the problems of mechanics of coupled fields, primarily to electro-magneto-elasticity, caused by the needs of today's technological advances in various industries and the development of innovation technologies. The issues of motion of a continuum with electromagnetic effects fill a highly important place in the mechanics of coupled fields.

One of the main directions of development of modern solid mechanics is a development of the theory of conjugate fields and, in particular, the theory of the electromagnetic interaction with deformable medium[8, 10, 11-13, 14-18, 23, 24].

The mechanism of interaction of an elastic medium with the electromagnetic field is diverse and depends on the geometrical characteristics and physical properties of the body under consideration.

In particular, this mechanism gets some specifics when considering the problems of thin plates and shells having anisotropic conductivity.

In creating optimal structures in modern engineering, widespread use is made of thin-walled shells and plates as structural elements in which effects of nonlinear electromagnetic interaction with magnetic fields are significant.

Effects of the coupling of mechanical displacements of conductive bodies with the electromagnetic field are conditioned by the Lorentz ponderomotive forces.

The Lorentz forces depend on the speed of the conductive elements of a continuous medium and the external magnetic field, the magnitude and orientation of the conduction current in reference to external magnetic field.

Significant effects of ponderomotive interactions occur for high frequency oscillations at large amplitudes of displacements, pulsed magnetic fields and current-carrying elements.

It is for these conditions are first necessary to develop mathematical foundations of magneto elasticity and applied methods for solving certain classes of problems.

Thin shells are widely used as members of advanced structures.

Due to more stringent requirements to the service conditions of such structures, not only rigid but also flexible shells should be used[2, 6, 7, 25].

Along with the development of the theory of flexible shells, it is also necessary to develop the theory of flexible anisotropic shells in the non-stationary magnetic fields[3, 19, 20, 24].

Problems interaction between electro-magnetic field and deformed bodies are frequent in advanced technology.

II. NONLINEAR FORMULATION OF THE PROBLEM. BASIC EQUATIONS

Flexible current-carrying conical shells of variable thickness, finite conductivity, excluding the effects of polarization and magnetization and thermal stresses are considered. Elastic properties of the shell are considered orthotropic, which main directions of elasticity coincide with the directions of the corresponding coordinate lines. Material obeys the generalized Hooke's law and has a finite conductivity. Electromagnetic properties of the material of the current-carrying shell are characterized by tensors of electrical conductivity $\sigma_{ii}$, magnetic permeability $\mu_{ij}$ and dielectric permittivity $\varepsilon_{ij} (i, j=1,2,3)$.

At the same time due to the crystallophysics for the considered class of conducting media with rhombic crystal structure it was considered that the tensors $\sigma_{ij}, \mu_{ij}, \varepsilon_{ij}$ take a diagonal form[4, 15, 22].

Note that in this case, an arbitrary surface of the second order has three mutually perpendicular axes and the second order can be positioned axis parallel to the crystallographic axes of the second order and second order surface characteristic has all symmetry elements that may be in the orthorhombic system classes.

Coordinate surface of current-carrying orthotropic shell in the unreformed state we assign to curvilinear orthogonal coordinate system $\alpha, \beta, \gamma$ assuming that the coordinate lines of the middle surface of the shell coincide with the lines of main curvatures.
Suppose that the geometrical and mechanical characteristics of the body are such that to describe the deformation process is applicable version of the geometrically nonlinear theory of thin shells in the quadratic approximation.

Also we assume that the relative strength of the electric field $\hat{E}$ and magnetic field $\hat{H}$ is performed electromagnetic hypothesis\[1, 3, 5\].

$$E_i = E_i(\alpha, \beta, t); E_z = E_z(\alpha, \beta, t); E_z = -\frac{\partial B_z}{\partial t};$$

$$J_i = J_i(\alpha, \beta, t); J_z = J_z(\alpha, \beta, t); J_z = 0;$$

$$H_z = \frac{1}{2} \left( H_z^T + H_z^+ \right) + \frac{1}{h} \left( H_z^T - H_z^+ \right);$$

$$H_z = \frac{1}{2} \left( H_z^T + H_z^+ \right) + \frac{1}{h} \left( H_z^T - H_z^+ \right); H_z = H_z(\alpha, \beta, t).$$

where the $u_i$ components of the displacement vector envelope planes; $E_i, H_i$ components of the vectors of the electric and magnetic fields shell; $J_i$ eddy current components; $H^z$ - the tangential components of the magnetic field on the surface of the shell strength; $h$ - thickness of the shell.

These assumptions are some electrodynamic analog of the hypothesis of non-deformable normals and together with the latter hypothesis magnetoelasticity makes subtle bodies. The adoption of these hypotheses allows us to reduce the problem of the three-dimensional deformation of the body to the problem of deformation of the chosen arbitrarily coordinate surface.

To develop techniques for the numerical solution of the new class of related problems of the theory of orthotropic magnetoelastic conical shells of revolution having orthotropic conductivity based on the consistent application of the finite Newmark schemes\[21\], linearization method and discrete orthogonalization \[10, 14, 15\].

To make effective use of the proposed methods supposed that the appearance of an external magnetic field does not appear sharp skin effects on the thickness of the shell and an electromagnetic process coordinate quickly enters the mode close to steady.

This leads to restrictions on the behavior of the external magnetic field and on the geometric and electrical parameters of the shell

$$\frac{\tau}{h^2 \sigma \mu} > 1,$$

where the $\tau$ - characteristic time of the magnetic field. In case of failure to do so should be considered only the shell of the equation of motion by the magnetic pressure.

In this formulation, the system of equations describing the time in the appropriate layer of nonlinear oscillations of a flexible current-carrying orthotropic conical shell of variable thickness, according, after application of the method takes the form quasi-linearization.


\[ \frac{dB_z^{(1\pm)}}{dm} = -\sigma_2 \mu \left[ E_\theta^{(1\pm)} + 0.5 \left( \nu^{(1\pm\Delta)} \right) \delta^{(1\pm)} \left( B_z^{(1)} + B_z^{(2)} \right) - \left( \delta^{(1\pm\Delta)} \right) \delta^{(1\pm)} B_z^{(3)} + \left( \delta^{(1\pm\Delta)} \right) \delta^{(1\pm)} B_z^{(4)} \right] + \frac{B_z^{(3)} - B_z^{(4)}}{\mu \rho} ; \]

\[ \frac{dE_\theta^{(1\pm)}}{dm} = -\frac{1}{\rho} \left( B_z^{(1\pm\Delta)} \right) \delta^{(1\pm)} - \cos \phi \frac{E_\theta^{(1\pm)}}{\rho} \left( k = 0, 1, 2, \ldots \right). \]

In this case, the boundary conditions can be written as

\[ u = 0, \quad w = 0, \quad M_s = 0, \quad B_z = 0.3 \sin \omega t \]

at \( s = s_0 = 0 \),

\[ w = 0, \quad \theta_s = 0, \quad N_s = 0, \quad B_z = 0 \]

at \( s = s_N = 0.5 m \). \hspace{1cm} (4)

The initial conditions take the form

\[ \bar{N}(s,t) \big|_{t=0} = 0, \quad \bar{u}(s,t) \big|_{t=0} = 0, \quad \bar{w}(s,t) \big|_{t=0} = 0 \]

\hspace{1cm} (5)

There \( N_1 \), \( N_2 \) - meridional and circumferential force; \( S \) - a shearing force; \( Q_n \) - cutting force; \( M_s, M_\theta \) - curving moments; \( u, \ w \) - displacement and deflection; \( \theta_s \) - the rotation angle of the normal; \( P_s, P_\theta \) - mechanical load components; \( E_\theta \) - circumferential component of the electric field intensity; \( B_z^+ \) - the normal component of the magnetic induction; \( B_z^- \) - known components of the magnetic induction on the surface of the shell; \( J_{ss} \) - component of the electric current density from an external source; \( e_s, e_\theta \) - elastic moduli in the directions; \( s, \theta \) - respectively; \( V_s, V_\theta \) - Poisson coefficients, characterizing transverse tensile compression direction of the coordinate axes; \( \mu \) - magnetic permeability; \( \omega \) - the angular frequency; \( \sigma_1, \sigma_2, \sigma_3 \) - the main components of the tensor conductivity.

Solving boundary value problems magnetoelectricity theory of thin shells with finite electrical conductivity in nonlinear formulation it is associated with great computational difficulties.

This is explained by the fact that the system, describing the stress-strain state of the shell-related that is composed of the equations of motion and electrodynamics. The equations of motion present volume Lorentz force, and the equations of electrodynamics include derivatives of the displacements over time. In addition, it is a nonlinear mixed hyperbolic-parabolic system of differential equations in partial derivatives of the eighth order with variable coefficients. The bulk of the Lorentz force-nonlinear and vary depending on the deformation of the middle surface of the shell and change the time coordinate.

III. A NUMERICAL EXAMPLE. ANALYSIS OF THE RESULTS

Consider the nonlinear behavior orthotropic current carrying conical shell of beryllium variable thickness that varies in the meridional direction of the law

\[ h = 5 \cdot 10^{-4} \left(1 - \alpha \frac{s}{s_N} \right)^m. \]

We believe that the skin is exposed to mechanical force

\[ P_s = 5 \cdot 10^3 \sin \omega t \frac{N}{m^2}, \]

and the external magnetic field \( B_{s0} = 0.1 T \) and that the envelope has a finite conductivity orthotropic.

We assume that by the electric current in the disturbed state is evenly distributed on the shell, the external current density does not depend on the coordinates. In this case, the combined effect on the shell loading, the ponderomotive force consisting of Lorentz forces and mechanical. Contour small radius \( s = s_h \), hinged, and the second path \( s = s_N \) - free in the meridional direction.

We investigate the behavior of orthotropic shell, depending on the changes in shell thickness. The problem for orthotropic cone of beryllium variable thickness \( h = 5 \cdot 10^{-4} \left(1 - \alpha \frac{s}{s_N} \right)^m \) calculated for different values of the parameter \( \alpha = \{0.2; 0.3; 0.4; 0.5\} \) characterizing the variability of thickness in the meridional direction.

The parameters of the shell and the material are:

\[ s_0 = 0, \quad s_N = 0.5 m, \quad h = 5 \cdot 10^{-4} \left(1 - \alpha \frac{s}{s_N} \right)^m, \]

\[ r = r_0 + s \cos \phi, \quad r_0 = 0.5 m, \quad \omega = 314.16 \text{sec}^{-1} \]

\[ \rho = 2300 kg/m^3, \quad B_z^+ = B_z^- = 0.5 T, \quad \varphi = 30' \]

\[ B_{s0} = 0.1 T, \quad \mu = 1.256 \cdot 10^6 H/m \]

\[ J_{ss} = -5 \cdot 10^5 \sin \omega t A/m^2, \quad \sigma_1 = 0.279 \cdot 10^8 (\Omega \cdot m)^{-1}, \]

\[ \sigma_2 = 0.321 \cdot 10^8 (\Omega \cdot m)^{-1}, \quad \sigma_3 = 1.136 \cdot 10^8 (\Omega \cdot m)^{-1}, \]

\[ V_s = 0.03, \quad \nu_\theta = 0.09, \quad P_s = 5 \cdot 10^3 \sin \omega t N/m^2, \]

\[ e_s = 28.8 \cdot 10^9 N/m^2, \quad e_\theta = 33.53 \cdot 10^9 N/m^2 \]

The solution is found in the time interval \( \tau = 0 \div 1 \cdot 10^{-2} \text{sec} \) for the integration time step is chosen to be \( \Delta t = 1 \cdot 10^{-3} \text{sec} \). Consider the case in the anisotropy of the electrical resistance equal to beryllium \( \eta_s / \eta_h = 4.07 \). In the following graph the figures (1, 2, 3, 4) correspond to the values of the parameter \( \alpha = \{0.2; 0.3; 0.4; 0.5\} \).

Figure 1 shows the distribution of deflection of the shell along the meridian at a time \( t = 5 \cdot 10^{-3} \) for various values of \( \alpha \).
It was found that the maximum deflection along the shell arises about the meaning of the neighborhood. This is due to the fact that according to the boundary conditions of the left end is hinged and the right free end of the sheet in the meridional direction. In addition, the shell thickness ranging from the left end to the right end face is reduced to 2 times at $\alpha = 0.5$. Therefore, the maximum values of deflections occur near the right end of the shell.

When taking into account the effect of the thickness of the stress cone shell was regarded as the amount of stress and Maxwell stress, consider the total stressed state.

Figures 2 and 3 shows the distribution of the maximum stress values ($\sigma_{zz} + T_{zz}^+$) and ($\sigma_{zz} - T_{zz}$) along the meridian of the shell at the time $t = 5 \cdot 10^{-3} s$ on the outer and inner surfaces of the shell for different parameter values $\alpha$.

Curves 1-4 characterize the stress distribution for the corresponding parameter values $\alpha$.

The figures demonstrate the complexity of the behavior of the shell, depending on the boundary conditions under the influence of mechanical and magnetic fields.

It should be noted that the maximum value is observed in all cases $s = 0.5 m$ and with the parameter $\alpha$- the voltage values on the surfaces of the shell increases.

Figures 4 and 5 show the rates of change ($\partial u / \partial t$) or acceleration ($\partial^2 u / \partial t^2$) movable along a meridian of the shell for the different time $t = 5 \cdot 10^{-3} s$ parameter values $\alpha$.

From the figures it is clear that with increasing parameter value $\alpha$ and accordingly, a decrease in $h(s)$ thickness, is an increase in the speed of longitudinal movement along the meridian.

IV. Conclusion

In this article, the associated task magnetoelasticity for flexible orthotropic conical shell taking into account the orthotropic conductivity. The effect of thickness on the stress-strain state of orthotropic shell.

The results are indicative of the effect of shell thickness on the deformation and the need to take account of this factor in the calculation scheme.
As can be seen, of variable thickness has a significant impact on the change in the stress-strain state of the shell, and account geometric nonlinearity allows to specify a picture of deformation.

REFERENCES
