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FEA and Spectral Analysis for Viscoelastic Structures in Frequency and Time Domain

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***Abstract* – Rubber bushing installed on the suspension system of vehicles plays a critical role of vibration isolator and interface to transmitting energy. A finite element method is developed to analyze the dynamic response of rubber bushing in the frequency domain. The time domain response of viscoelastic structure is realized simply by utilizing the spectral analysis. With this approach, the creep of viscoelastic models is evaluated and the standard linear model of viscoelastic material is employed in this article to represent the constitutive relationship of rubber. The FEA frequency scan of rubber bushing is verified through the experimental modal testing.**

***Keywords* – FEA, Dynamic, Viscoelastic, Spectral Analysis.**

I. INTRODUCTION

Rubber bushing work as a connection component, it can relate the small parts such as tie bar and lower arm with the full vehicle. It is also an isolator aiming at minimizing the transmission of excitation force or displacement from a source to receiver [1]. Viscoelastic damping material can effectively reduce the noise in lightweight vehicles and vibration [2]. The passive damping composed of viscoelastic material is also widely used to control the interior sound quality in aerospace [3]. A typical structure of vibration-proof rubber bushings is a hollow elastomer cylinder, which is surrounded by inner and outer cylindrical steel sleeves which possesses high loss factor and high durability[4]. Rubber has good resilience and energy storage capability. Working as a spring, rubber exhibits the ability to recover from a large deformation to original geometry even at repeated cycling. In extreme working case, the ultimate elongation of rubber can reach 1000% of the original length and the corresponding stress of that elongation is still pretty low because of the low modulus of rubber [5].

Time-dependent behavior of rubber bushing is complex, which embodies both shear and volumetric deformation. Complex mixtures of an elastomer base resin, particulate fillers like carbon black and/or fumed silica, etc, as well as the part of manufacturing process affects the performance of the engineering rubber. Slight change of rubber supply, formulations may affect the performance of rubber, while those changes may not attract the attention of suppliers. The interactions between the filler and the polymer matrix and also the interactions within the fillers lead to a large shear modulus magnitude. The mechanism of frictional behavior, breaking of filler structure and loss factor have been characterized by several authors[6, 7]. The dynamic application of bushing is complex because of the loading histories, which includes both step/impulse transient loading and multi-frequency harmonic loading. Thus, it is difficult to predict the effect of complex loading histories on the nonlinear, time-dependent material[8].

To model the dynamic response of bushing, there are two basic approaches[9]. The first approach utilizes the constitutive equation of rubber material to calculate the strain-displacement of any designed bushing. The second approach is to determine the force-displacement relation experimentally. The first approach is used mostly to analyze the structure composed of viscoelastic material, especially with the development of FEA. For the nonlinear viscoelastic material, the integral constitutive law or differential law is proposed to describe the material behaviors and the algorithm is developed in the time domain to calculate the response at each time step[10]. The time-based fractional calculus method was investigated to predict the viscoelastic behavior using the four-parameter fractional solid model[11]. A simple and direct approach to develop the FEA coding that predicts the force-displacement of rubber components is to start from the linear viscoelastic material model. Maxwell model proposed by James Clerk Maxwell [12] and Vogit model [13] are the simplest two constitutive models to characterize the elasticity and viscoelasticity of rubber. The derivative of strain over time is time dependent, while the linear time-dependent system is recommended to be considered in the frequency domain[14]. This article is focused on the exploration of FEA approach based on the linear viscoelastic material model in order to deal with the dynamic simulation in frequency domain. The development of the viscoelastic matrix is the challenge in FEA and Golla [15] explored the application though the introduction of "dissipation" coordinates to include viscoelastic damping. In this article, the frequency dependent complex stiffness matrix is developed to solve the dynamic response of structures composed of viscoelastic materials.

Most literatures about viscoelastic structures discussed the dynamic behavior of rubber in time or frequency domain independently and exclusively. Wave propagation in elastic structure can be built based on the transfer function in frequency domain and reconstructed in time domain using spectral analysis [16]. Forced harmonic excitation is merely an extreme loading case applied on the rubber components used in automobile and aerospace. The developed FEA model can easily perform the single frequency or frequency scan calculation. The implement of spectral analysis enable the time domain input and output process more efficient and effective. In fact, this approach can throw a new light on the development of viscoelastic FEA, which is mostly confined in the time domain constitutive relationships. In this article, the developed force-frequency module is termed as Simplex to differentiate from the elastic modules in QED. QED is a user friendly interface of elastic structure analysis incorporating the mesh generation and simulation result views.

The structure of this article is listed as follows. Part 2 introduces the spectral analysis and its advantage in conducting analysis of viscoelastic structure. The creep of three classical constitutive models demonstrates the FFT and IFFT reconstruction in dealing with time domain problems. Part 3 presents the approach to develop the FEA program for frequency domain analysis and its application on elastic structure with damping. Part 4 encompasses the viscoelastic into the stiffness matrix of the governing equation and frequency scan results of bushing composed of rubber and steel are compared with the natural frequency measured from hammer impact experiment. The response of blast load applied on the viscoelastic structure indicates the capability of switching the FEA into time domain and potential application in dealing with the wave propagation in viscoelastic structure.

II. The Viscoelastic Constitutive Model and Spectral Analysis

The standard liner model is composed of two branches in parallel. Branch one is in series connected spring element k_1 and dashpot η and the branch two is the spring element k_2. The constitutive relationship is time dependent and written as,

$$k_2\varepsilon + (k_1 + k_2)\tau \, {}^{d\varepsilon}/_{dt} = \tau \, {}^{d\sigma}/_{dt} + \sigma \tag{1}$$

Where $\tau = {}^{\eta}/_{k_1}$, in the lower frequency range, the spring and dashpot coefficient are k_1=0.8MPa, k_2=1.07MPa and η=0.0303MPa. Those parameters are identified from the dynamic mechanical analyze (DMA) testing at different frequencies and amplitudes[17]. The constitutive relationship of the viscoelastic model is a differential equation, which posts a lot of challenges to integrating them in order to get the force-displacement or stress-strain relationship in the time domain. From the definition of transfer function and the hereditary of viscoelastic material, the displacement history of the viscoelastic structure can be represented as,

$$U(t) = \int_0^t G(t - t^*)P(t^*)dt^* \tag{2}$$

Where t^* is the elapsed time, $P(t^*)$ is the loading at time t^* and $G(t - t^*)$ is the transfer function which is time dependent. The kernel of the integral is the time dependent transfer function G.

In order to have the response at time t, it is necessary to do the integral over all elapsed time. Similarly, the response at $t + dt$ needs the sum up of product of $G(t - t^*)P(t^*)$ from the initial time to the current time. That means the response at any time needs the calculation of the integral over all elapsed time. If the time domain in (2) includes 1024 points, then a 1024 times integral should be completed in order to get the whole response in the time domain. That approach is very complicated and inefficient since it introduces a lot of calculation.

Alternatively, there are actually many more powerful skills to solve the integral, one of most efficient approach is the spectral analysis using the FFT algorithm. The fundamental method to solve the convolution is to do the Laplace transform, which transforms the integral into frequency domain and then do the product. The transformation from time domain to frequency domain implements the FFT and IFFT algorithm, which is very efficient. This approach saves a lot of steps since no summation is needed expect for the product in frequency domain. The response of each point in the time domain can be obtained after the inverse IFFT. It is easiest to work in the frequency domain when the structure is made of linear viscoelastic materials. Some problems begin in frequency domain and end in the frequency domain, such as sinusoidal vibration. For those problems, it is easier to understand the frequency response in single or frequency scan. While some of the problems begin in the time domain and end in the time domain, such as the creep and blast loading. The periodic and single frequency excitation is an extreme loading case in the real working environment of rubber bushing. Thus, spectral analysis is desired and applied to deal with the vibration problem under general loading history in time domain. With the Fourier analysis, any signal in time domain can be represented as multiple harmonic signals at different frequencies. The loading and displacement represented using spectral analysis is written as,

$$P(t) = \sum_n \hat{p}_n e^{i\omega_n t}, \quad u(t) = \sum_n \hat{u}_n e^{i\omega_n t} \tag{3}$$

Then, the loading and displacement in the governing equation can be connected using the transfer function at each frequency components.

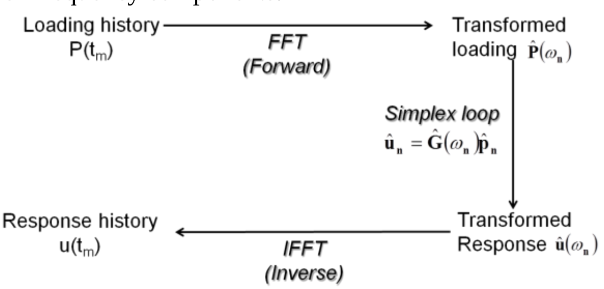

Fig.1. The flowchart of using spectral analysis in general time domain problem.

The flowchart in Fig. 1 shows the process to tackle the vibration problem in time domain. The work in the flowchart includes three steps. The first step is to transform the loading history of the structure from time domain into frequency domain using the FFT algorithm. The second step is to encompass the constitutive relationship of the material model into the governing equation of the FEA. In this testing, the linear viscoelastic material is chosen and their differential constitutive equations are equally represented using the frequency function of modulus. Frequency dependent modulus contributes to understand the material properties of the rubber and facilities the dynamic analysis in the frequency domain, especially the linear dynamic analysis. In step two, a frequency loop is developed to calculate the response of the structure at each frequency [16],

$$\hat{u}_n = \hat{G}(\omega_n)\hat{P}_n \quad \text{or} \quad \hat{\sigma}_n = \hat{G}(\omega_n)\hat{\varepsilon}_n \tag{4}$$

$\hat{G}(\omega_n)$ is the transfer function of the system in frequency domain. The third step is to get the response in time domain after reconstruction using IFFT algorithm.

Fig. 2 shows the creep behavior of three viscoelastic model when the step stress is applied. The solid model in Fig. 2 is Vogid model and the liquid model is Maxwell model. In the current test, N is set as 8192 and time step dt is set as 0.1 to balance the accuracy and burden of computation. Here, the stress-strain relationship is represented with the transfer function $\hat{G}(\omega_n)$, which is shown in (5),

$$\hat{G}_{solid} = \frac{1}{(\hat{E} + i\omega\eta)};\ \hat{G}_{liquid} = \left(\frac{i\omega}{\hat{E}} + \frac{1}{\eta}\right);$$

$$\hat{G}_{stan} = \frac{(\tau i\omega + 1)}{(E_2 + (E_1 + E_2)\tau i\omega)} \tag{5}$$

The rise and drop time of loading is 1s and the stress 1MPa is applied for 2, 20, 200s to investigate the time scale of creep in the three types of viscoelastic models. With the proper selection of spring and dashpot coefficients, the creep happens in the initial 20s of stress loading. After the strain reaches the maximum, further stress loading has no effect on the strain in the solid model. However, the strain of the liquid model gives a proportional increase with the stress duration, which is inappropriate to describe a solid rubber. It is important to claim that $\hat{G}_{liquid}$ is actually the transfer function between stress and strain rate since the reconstruction of strain in liquid model couldn't guarantee the periodicity of the FFT and IFFT. Thus, the strain is obtained from $\varepsilon = \int \dot{\varepsilon}\mathbf{dt}$ after the reconstruction of strain rate. Fig. 2(d) shows that there is no linear increase of strain in the solid model. Thus, the comparison confirms the standard linear model as the simplest viable one to develop the viscoelastic FEA program.

II. Numerical Solution and Application in Elastic Structure with Damping

In reality, damping or energy dissipation can be found in almost all system in motion. The governing equation of motion or the force equilibrium of a system is,

$$M\ddot{u} + X\dot{u} + Ku = P(t) \tag{6}$$

Where $M\ddot{u}$ is the inertia force, $C\dot{u}$ is the damping force, Ku is the elastic force and $p(t)$ is the applied loading force. From the field test simulating the external excitation applied on the rubber bushing installed on the exhaust piping system, the vibration in big time window can be approximated using single frequency excitation. In that extreme case, the particular loading can be characterized as sinusoidal or cosine function. Under harmonic excitation, the force equilibrium of the damped system can be written as,

$$[K]v + [C]\dot{v} + [M]\ddot{v} = Pcos(\omega\mathbf{t}) \tag{7}$$

The complementary force equilibrium of the system is,

$$[K]w + [C]\dot{w} + [M]\ddot{w} = Psin(\omega\mathbf{t}) \tag{8}$$

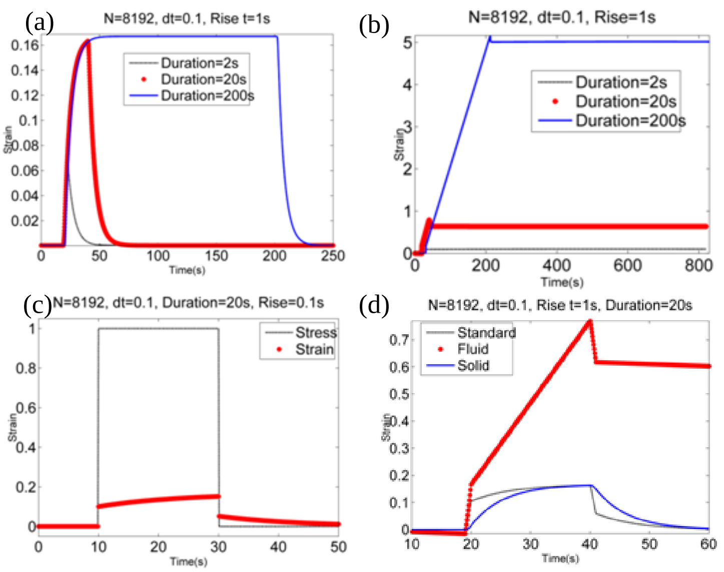

Fig. 2 Creep behaviors: (a) Solid model; (b) liquid model; (c) Standard linear model; (d) compassion of three models

All those variables in the two equations are real and the only difference is the loading phase. Combine the two linear systems with the definition of complex displacement, $u = v + iw$, then, the new created governing equation turns to,

$$[K]u+[C]\dot{u}+[M]\ddot{u}=Pe^{i\omega t} \qquad (9)$$

The complex displacement can also be written as $u=\hat{u}e^{i\omega t}$. The system of the equation is taken as pseudo-static as the loading is periodical. In the damped system, it is tedious to solve the equation in the function of sinusoidal, cosine or exponential form. Especially, for a system with time differential governing equation, the introduction of complex quantities can greatly simplify the solving process. After remove the exponential term at two sides, the equation turns to frequency dependent, which explains the variation of response at different excitation frequency,

$$\left|[K]+i\omega[C]-\omega^2[M]\right|\hat{u}=P \qquad (10)$$

The left term in the bracket is the dynamic stiffness matrix $[\hat{K}_D]$. In the Simplex program, the structure damping of elastic material is generally characterized using the damping coefficient C. As to a system constructed using Rayleigh damping, the damping matrix is proportional to the structural stiffness and mass matrices,

$$[C]=\alpha[M]+\beta[K] \qquad (11)$$

Where, the α and β are constants. And the corresponding dynamic stiffness is given as,

$$\hat{K}(\omega)=[K]-\omega^2[M]+i\omega(\alpha[M]+\beta[K]) \qquad (12)$$

To elaborate the application of FEA and spectral analysis on the structure, the cylindrical structure made of two materials has been tested. The inner shaft is made of higher modulus material-steel, which is bonded with the lower modulus material-rubber. The cylindrical structure is defined as elastic structure with damping and both steel and rubber are characterized using Rayleigh damping. The outer steel sleeve confines the movement of the external surface of the rubber and imposes zero degree of freedom on those nodes located at the external surface considering the extremely large difference of modulus between steel and rubber. The mesh of the remaining steel and rubber is plotted in Fig. 3, where the sketch in blue is rubber and the sketch in yellow is steel shaft. Hex20 element and 27 point integration approach are employed in the pre-processing and post-processing of FEA program separately. The vertical harmonic excitations at different frequencies are applied on the steel shaft to simulation the working environment of the suspension system of vehicles.

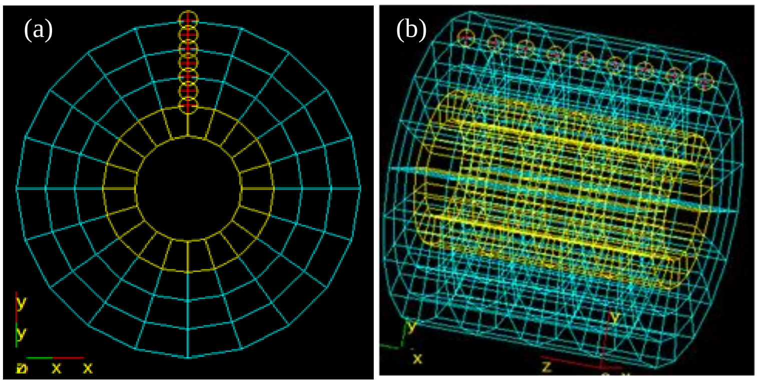

Fig. 3. Mesh of two layers cylindrical structure (a) Radial distribution of nodes; (b) axial distribution of nodes.

The nodes in radial direction shown in Fig. 3(a) are 21, 42, 77, 108, 124,174, and 213 ranging from bottom to the top and their vertical displacements are collected and compared. The nodes in axial direction shown in Fig. 3(b) are 124, 262, 427, 562, 724,862, 1025, 1162, and 1327 from left to the right. Before the frequency scan, the Eigen value vibration analysis has been carried out to identify the modal shape and undamped natural frequency of the structure using QED. The frequency scan results from Simplex are shown in Fig. 4, which gives the natural frequency 402.7HZ under vertical excitation.

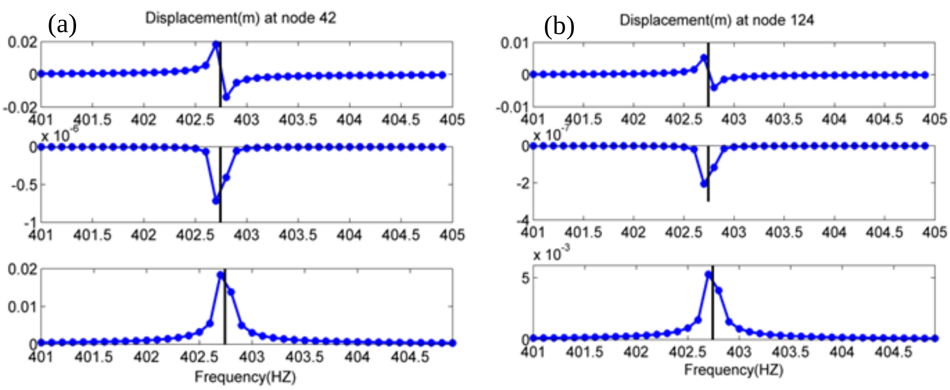

Fig. 4. Displacement of nodes under frequency scan (a) node 42; (b) node 124.

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The corresponding undamped natural frequency plotted in black lines is a little bit higher than the damped natural frequency. In this elastic structure with low damping, the imaginary part of the displacement is overwhelmed by the real part. Thus, the modal shape of the cylindrical structure under vertical load is dominated by the real part of the displacement. Because the node 124 is closer to the fixed external surface of the rubber core, its displacement is slightly lower than displacement at node 42.

The circumferential distribution of the displacement under the natural frequency is also collected to have the comprehensive understanding about the deformation of rubber core. Because of symmetry of the structure, only half of the nodes in a circle are analyzed. The nodes marked in Fig. 5 (a) are 75, 121, 122, 124, 126, 145, 146, 143, 147, 149, 152 and 153 ranging from left to right. Fig. 5(b) plots the distribution of displacement in circumference. Nodes locate at horizontal line and vertical lines have the maximum displacements compared with nodes at other positions.

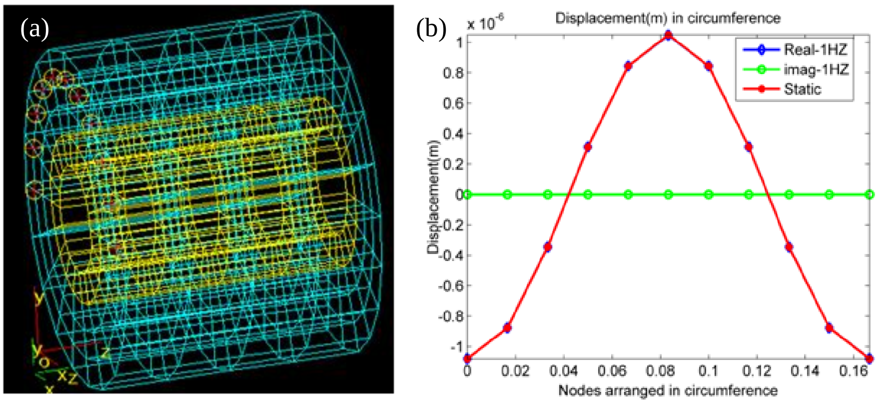

Fig.5. Displacement at first natural frequency (a) Nodes collected in circumference; (b) the circumferential distribution.

The stress distribution in radial, axial and circumferential directions is also plotted against the static stress distribution calculated from QED in Fig. 6. As to the radial and axial direction, the stress of loading direction is significant while the shear stress in circumferential direction is also attractive. The elastic structure implemented in the QED is of no damping and the current Simplex file is elastic structure with damping, thus, the Simplex generates complex stress and a scale is multiplied to match the QED results. Fig. 6(a) and (b) compares the stress σ_{yy} distribution at 1HZ and 100HZ. The overlap of real part stress and static calculation indicates the reliability of Simplex in dealing with complex variables. The stress at 100HZ is attributed to the higher displacement when the excitation frequency is closer to the natural frequency 402.7HZ.

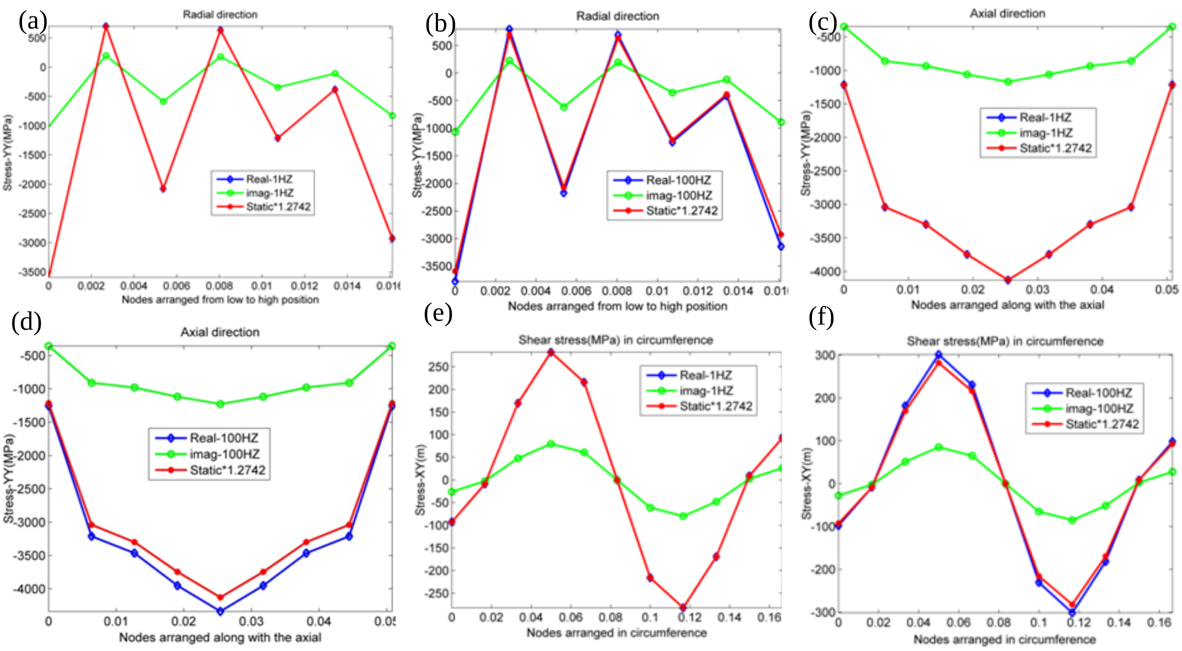

Fig.6. Stress distribution in (a) and (b) Radial; (c) and (d) axial; (e) and (f) circumferential.

Fig. 6(c) and (d) compare the stress σ_{yy} distribution in axial direction at frequency 1HZ and 100HZ. While as to the circumferential direction, the attractive shear stress σ_{xy} shown in Fig. 6(e) and (f) is compared. The three groups of comparison prove that with the increase of external excitation frequency, the magnitude of the stress increases slightly as the resonance effect on the displacement.

III. Dynamics of Viscoelastic Structure in Frequency Domain and Time Domain

Nevertheless, as to the viscoelastic structure, the damping inherited in the material overwhelms the structure damping, and thus, the latter is negligible in the following programming. As to the homogenous material in one model, each element has the same material tag. The frequency dependent modulus of the viscoelastic structure can be simulated by multiplying $\varphi(\omega)$ with the assembled dynamic stiffness matrix K. With the identified parameters in the constitutive equation of standard linear model, the stiffness matrix can be represented as,

$$\hat{\mathbf{K}}(\omega)=\left(\frac{\mathbf{i}\omega\eta(\mathbf{E}_1/\mathbf{E}_2+\mathbf{1})+\mathbf{E}_1}{(\mathbf{E}_1+\mathbf{i}\omega\eta)}\right)\mathbf{K}=\phi(\omega)\mathbf{K} \tag{13}$$

In the case of the complicated structure, for example, the sandwich cylindrical bushing composed of two materials, the boundary nodes belong to two materials and the K matrix is assembled based on the modulus of two materials, it is not recommended to multiply the unique $\varphi(\omega)$ with the K matrix. In that case, the coefficient $\varphi_1(\omega)$ and $\varphi_2(\omega)$ should be multiplied separately with the two materials before the assembly.

In the force-frequency subroutine, all input and output variables are in the frequency domain. The time domain loading history is implemented in the Simplex after FFT and the response in time domain can be reconstructed with IFFT from frequency domain. The Simplex FEA program deals with the displacement, strain, stress and energy dissipation in frequency domain for single frequency or frequency scan. Correspondingly, all outputs have imaginary part accompanying the real part as the modulus of the material is a complex variable.

The coefficient in this example is merely 0.1η to have more obvious modal shape in case the response is over damped. In theory, Poisson ratio (v) 0.5 or 0.495 should be assigned to this nearly incompressible material, however, 0.5 or close to 0.5 may cause trouble in the data processing during programming. To avoid the trouble, in Simplex, Poisson ratio (v) is set as 0.475. Figure 7 is the mesh of the viscoelastic beam and the geometry of the beam is 0.2m, 0.04m and 0.04m in length, width and height. The beam meshing includes 24 elements with six in length, two in width and two in height. The external harmonic loading is applied at the central line of the clamped beam at 1HZ and then do a frequency scan.

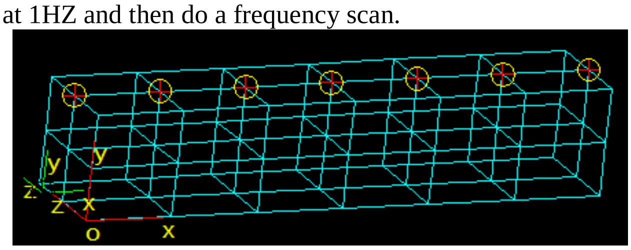

Fig.7. The sketch of beam clamped at two ends.

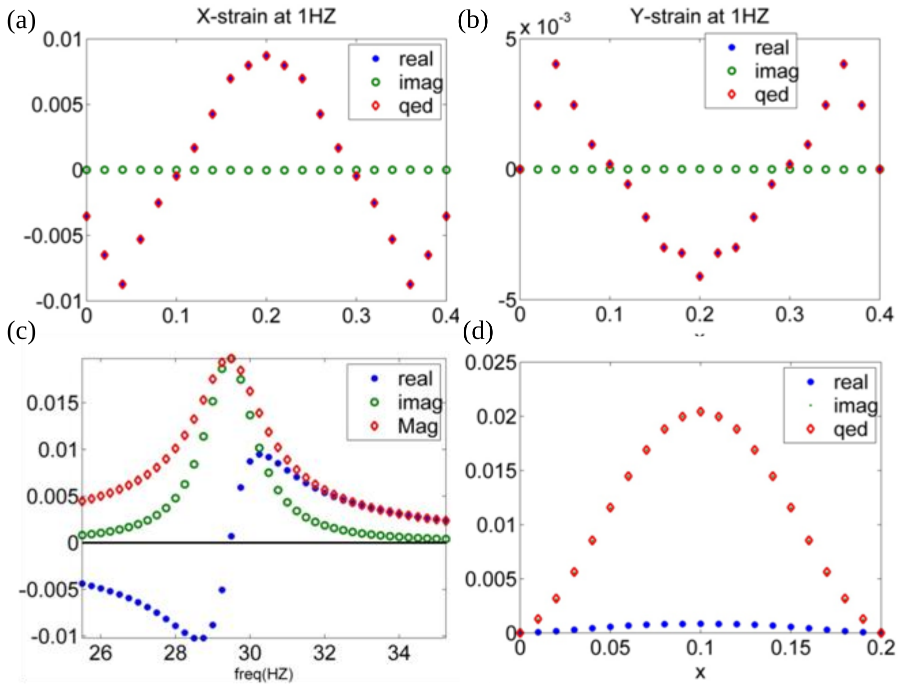

Fig. 8. (a) Strain in X; (b) strain in Y; (c) frequency scan; (d) the first modal shape.

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At lower frequency, the influence of imaginary part of the modulus is pretty trivial, it is reasonable to compare it with static deformation. The real part of the displacement plays the dominate role and the imaginary part is nearly negligible. The strain of beam calculated from Simplex nearly tallies with the QED under 1HZ excitation as shown in Figure 8 (a) and (b), which verifies the accuracy of the Simplex program in dealing with the viscoelastic structure. The maximum strain appears at the central part of the clamped beam and the strain is symmetrical over the central point.

From the response of the beam under the frequency scan, it is convenient to get the damped natural frequency of the clamped viscoelastic beam. Fig. 8(c) shows the first natural frequency 29.5HZ and the modulus at this frequency is 1.0761MPa. Fig. 8(b) shows the first modal shape, which also indicates that the imaginary part of the displacement overwhelms the real part of the displacement. Since QED gives the static deformation while Simplex presents the resonance deformation, a scale is applied to have the comparison. Nevertheless, the perfect match of the Simplex model shape with that from the QED still confirms the effectiveness of the Simplex encompassing the complex modulus.

Rubber's modulus is frequency dependent, and the displacement of the clamped beam is also frequency dependent corresponding. The two black lines are the first natural frequency of the purely elastic structure calculated from QED. The first natural frequency is 32.02HZ when the modulus in the QED is set as $E = E_2$ =1.07MPa. Again, the first natural frequency is 42.3593HZ when the modulus in the QED is set as $E = E_1 + E_2$ =1.87MPa. Fig. 9 indicates that the higher η increases the dynamic modulus and shifts the location of the peak to the right side. No matter the real part or imaginary part, the magnitude decreases with the increasing η at the beginning. But later, the increasing of η causes the increase of magnitude. In the frequency range 30-40HZ, the imaginary displacement dominates the magnitude, thus, the response calculated from the magnitude is close to the response calculated from the imaginary part.

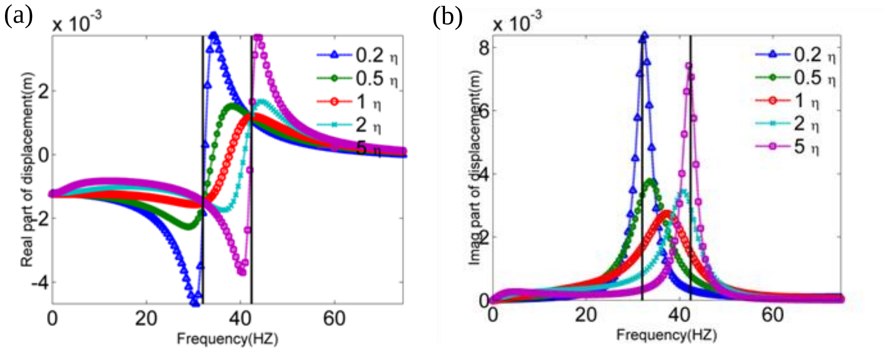

Fig. 9. The influence of damping coefficient on displacement at the central part of the beam (a) Real; (b) Image.

The hammer impact modal testing can be used to derive the transfer function of the rubber bushing. With the identified coefficients of transfer function, the energy dissipation of the rubber bushing at different excitation frequency can be predicted and the stability of the structure can be analyzed. However, for a large scale structure, the response measured at different position is varied, which means the transfer function between loading and displacement is also dependent on the measured location. During the modal testing, multiple accelerometers can be set up to record the response at different location. However, this approach is time consuming and redundancy, especially the repeated testing should be carried out each time for new designed rubber components. The Simplex program developed in frequency domain enables the user to evaluate the similar structures much easier and faster. The force-frequency Simplex program is actually a transfer function $\hat{G}(\omega_n)$ in frequency domain. If the unit amplitude harmonic excitation is applied on the structure, the output of any node on the structure stands for a unique transfer function.

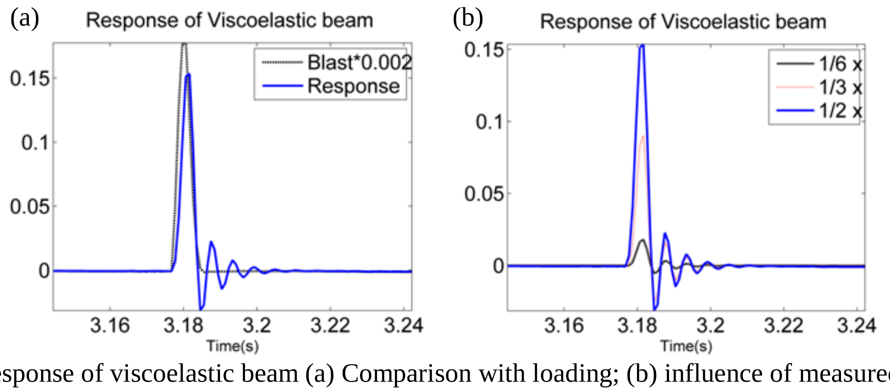

Fig.10. Response of viscoelastic beam (a) Comparison with loading; (b) influence of measured locations.

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To elaborate Simplex program in presenting the transfer function and the employment of spectral analysis, an impact loading is applied on the central of the clamped beam in the following discussion. Since $\hat{G}(\omega_n)$ is transfer function in frequency domain, the impact loading history should be transformed into frequency domain with FFT.

In this research, FFT is used rather than the Laplace transform due to the special requirement in coding. The calculated displacement from FEA is still in frequency domain, then, the inverse fast transformation (IFFT) is necessary to reconstruct the time history of the response.

Fig. 10(a) compares the displacement of response (solid line) with the impact loading (dash line). The impulse loading is multiplied with a scale 0.002 to have same magnitude as response in the plot. Because of the damping of rubber, the response of displacement is weakened in exponential form quickly. Fig. 10(b) compares the displacement at different locations of the clamped beam. 1/2 x is coordinates of the node located at the central of the beam, which displays maximum displacement. 1/6 x and 1/3 x are the distances from the fixed boundary of the beam to the measured nodes. Just as expected, the closer of the measured nodes to the fixed boundary, the lower is the peak value of the response. As to non-dissipative system, the response is similar to the loading history in time. However, as to the viscoelastic material, the transfer function is definitely dissipative and the response is different from the loading history. Fig. 10 also gives a proper demonstration of the possibility to investigate wave propagation in large structure with the current Simplex program and spectral analysis.

In the previous testing of Simplex, a frequency function multiplying with the dynamic stiffness of the elastic structure turns the stiffness into complex variable and frequency dependent in order to represent the stiffness matrix of viscoelastic structure. Whereas the precondition is the uniform material properties in all elements, which is inconsistent with the current rubber bushing composed of rubber and steel. To solve this problem, an approach is implemented to minimize the influence considering the modulus difference between rubber and steel. A group of comparison is listed in the Fig. 11 assuming the second material's modulus as frequency dependent. In this comparison, the damping coefficient is set as constant and spring coefficients are varied to simulate the stiffer second material. Fig. 11(b) indicates that with the increase of the two spring coefficients, the increase of storage modulus and loss modulus becomes slower. Especially, when the second material's spring coefficients are 1000 times of the rubber's, the frequency effect on modulus is almost negligible.

Since the modulus of steel is ten thousands time higher than that of rubber and the testing frequency is in the range of 0HZ to 100HZ, that means the modulus of steel is still constant in the testing range even though it is written as frequency function in Simplex. However, Fig. 11(c) shows that the second material's loss modulus increases linearly with the testing frequency. The further increase of spring coefficients has little change on the loss modulus behaviour as the three straight lines overlap with each other. To guarantee the applicability of the program in structure composed of two materials, another pre-condition is the negligible dissipation energy in the second material compared with that in the rubber even though the loss modulus is higher in the second material.

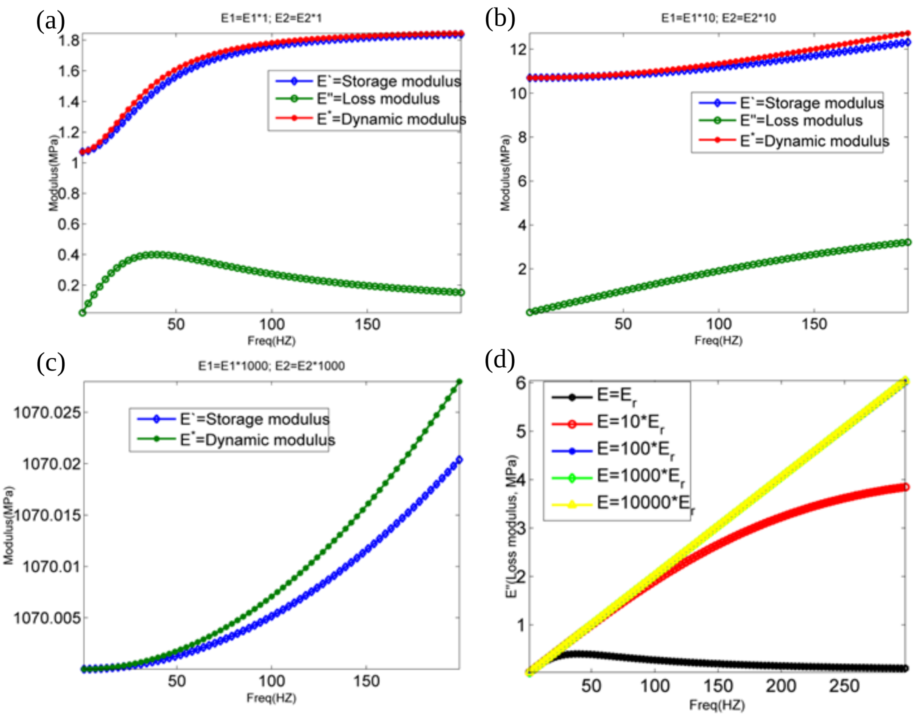

Fig. 11. Frequency response of material with different spring coefficients.

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Fig. 12 compares the dissipation energy distribution in the beam when the second material's spring coefficients are 10, 100, 1000, 10000 and 100000 times of the rubber's. For more straightforward demonstration, the ratio of dissipation energy in the two materials is compared, where E1 is the modulus of rubber and E2 is the modulus of the stiffer material. In this testing, the external excitation at frequency 30HZ is applied on the central top surface of the beam. Because the loss modulus of the second material increases with the frequency in the testing frequency range when the modulus difference between the two materials is not too large, the dissipation energy in the two materials are pretty close. However, once the two spring coefficients of the second material are extremely larger than that of rubber, for example, the steel in the Fig. 12(b), the ratio of dissipation energy between steel and rubber is as high as 14000 times. Thus, the dissipation energy in the steel is negligible compared with rubber and the Simplex program is applicable to structure composed with two materials as long as the second material's modulus is much higher than rubber's.

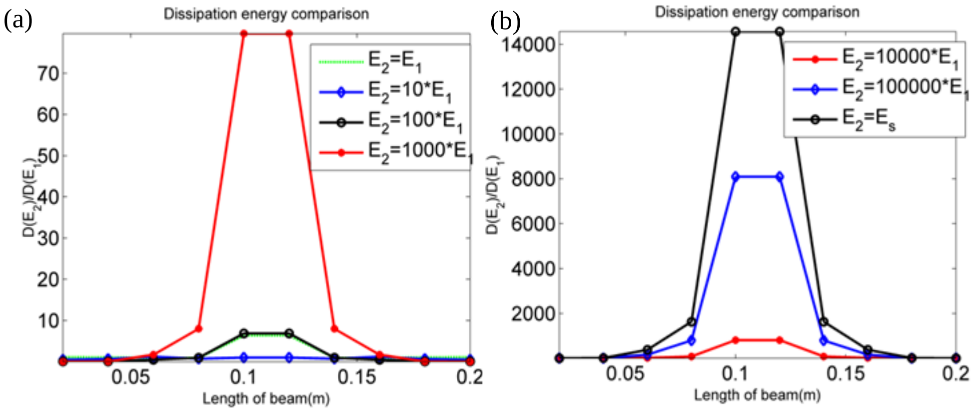

Fig. 12. Comparison of dissipation energy distribution at 30HZ in two materials.

Fig.13 displays the frequency scan of the beam composed of two materials. For the convenience of natural frequency identification in the viscoelastic structure, the Eigen values of the corresponding elastic structure are calculated using the low and high modulus separately, with the increase of the second material's low and high modulus, the first resonance frequency turns to higher. When both of the two layers materials are rubber, the frequency scan gives the first natural frequency 14.2HZ and the frequency plays dominate role to determine the displacement of the beam in the testing range 0HZ to 30HZ. Fig. 13 gives the frequency scan result of beam composed of rubber and steel. Because of the higher modulus of steel, the damped natural frequency appears at 850HZ, as shown in Fig. 13(b). However, besides of the frequency effect caused by the resonance, the frequency dependence of rubber materials displays in the testing range 0HZ to 100HZ. Because of the rising dynamic modulus of rubber in the low frequency range, the imaginary displacement reaches to a peak value before 100HZ and dominates the deformation of the beam.

Similar, the frequency scan is applied on the cylindrical structure composed of two materials. Fig. 14 compares the damped natural frequency with the natural frequency (black line) of the corresponding elastic structure, and the undamped natural frequency is always higher than the damped natural frequency. With the modulus increase of the second material, the first resonance frequency turns higher. Especially, when the second material is steel, in the high frequency range, the resonance affects the behaviour of the structure, but in the low frequency range, the viscoelastic of rubber affects the behaviour of the structure.

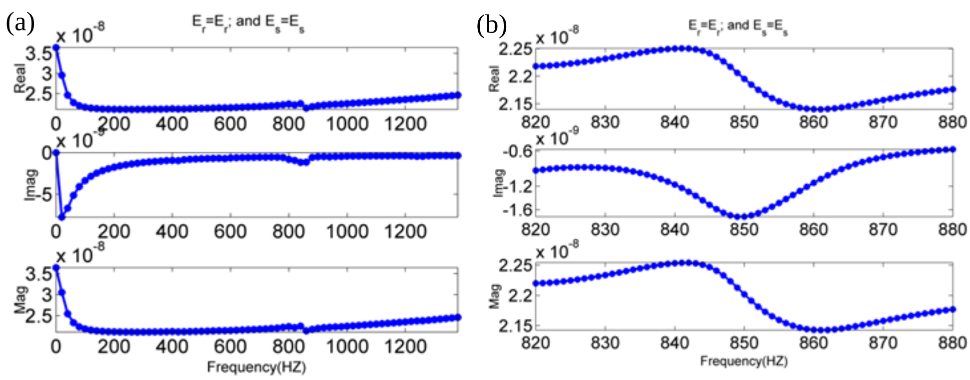

Fig.13. Frequency scan of the beam composed of two materials.

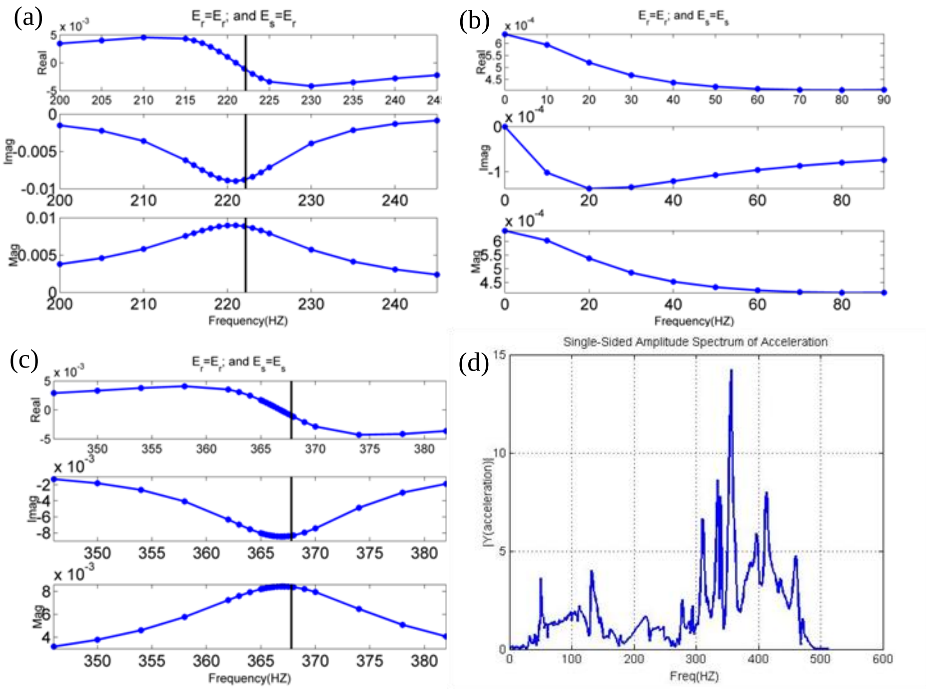

Fig.14. Frequency scan of viscoelastic cylindrical structure and FFT.

This characteristic is unique in structure composed with viscoelastic material since the frequency effect at low frequency is negligible in elastics structure. The frequencies scan in Fig. 14(c) gives the damped natural frequency of rubber bushing in axial at about 367HZ. The FFT of acceleration response from hammer impact testing in axial shows in Fig. 14(d) and indicates the natural frequency 355HZ. That calculated natural frequency from Simplex is rather close to the experimental measurement and that comparison verifies the feasibility of the simply and directly treatment about the steel in the force-frequency Simplex program.

CONCLUSION

In consideration of the complicated characterization of the nonlinear mechanical properties of the viscoelastic material and the barely satisfactory simulation of the commercial software, the finite element analysis is developed in this article to explore the dynamic response of complicated structure composed of viscoelastic materials in the frequency domain and time domain. With the spectral analysis, time domain response of structure composed of viscoelastic material becomes feasible. As a demonstration, the transfer functions of classical viscoelastic models are also elaborated and their merits and shortcomings are analyzed with the comparison of creep behaviours. The clamped beam is practiced in the testing process to verify the reliability of the Simplex program through the comparison with the quasi-static simulation. The blast load is investigated to testify the application of Simplex and spectral analysis in the viscoelastic structure wave propagation. Even though the research target of this article is dynamic analysis of rubber bushing, the developed Simplex program can be applied to rubber components under various working environments.

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REFERENCES

[1] J. Kadlowec, A. Wineman, and G. Hulbert, "Elastomer bushing response: experiments and finite element modeling". *Acta mechanica*, 163(1-2). 2003. pp. 25-38.

[2] R.Fan, G. Meng, J. Yang, and C. He, "Experimental study of the effect of viscoelastic damping materials on noise and vibration reduction within railway vehicles". *Journal of Sound and Vibration*, 319(1). 2009. pp. 58-76.

[3] Rao, Mohan D,"Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes". *Journal of Sound and Vibration*, 262(3).2003. pp. 457-474.

[4] N.Yoda, Y. Fujiwara, and S. Tanuma, Kinugawa Rubber Industrial Co., Ltd., 1992. *Structure of vibration-proof rubber bushing*. U.S. Patent 5,080,332.

[5] C. Lewitzke, and L. Ping," Application of elastomeric components for noise and vibration isolation in the automotive industry". *SAE transactions*, 110(6), 2001. pp. 1649-1669.

[6] A.Medalia, "Effect of carbon black on dynamic properties of rubber vulcanizates". *Rubber Chemistry and Technology*, 51(3),1978, pp. 437-523.

[7] G.Kraus, "*Reinforcement of elastomers by particulate fillers*". Science and technology of rubber, Academic Press, New York, 1978, pp. 387-416.

[8] A.Wineman, "Nonlinear viscoelastic solids—a review". *Mathematics and mechanics of solids*, 14(3), 2009, pp. 300-366.

[9] S.B. Lee and A. Wineman, "A model for non-linear viscoelastic coupled mode response of an elastomeric bushing". *International journal of non-linear mechanics*, 35(2).2000.pp. 177-199.

[10] M. Henriksen, "Nonlinear viscoelastic stress analysis—a finite element approach". *Computers & structures*, 18(1).1984. pp. 133-139.

[11] S. wJ Welch, R.A. Rorrer, and R.G. Duren Jr, "Application of time-based fractional calculus methods to viscoelastic creep and stress relaxation of materials". *Mechanics of Time-Dependent Materials*, 3(3).1999.pp. 279-303.

[12] D. Roylance, "*Engineering viscoelasticity*". Department of Materials Science and Engineering–Massachusetts Institute of Technology, Cambridge MA, 2139.2001. pp. 1-37.

[13] M. Meyers, and K. Chawla, "Mechanical behavior of materials", 1999. Prentice-Hall, Englewood Cliffs (NJ), see also Kallend JS, Kocks UF, Rollett AD, Wenk HR. Operational texture analysis. *Mater Sci Eng*, 132.1991. pp. 1-11.

[14] J.F.Doyle, "Viscoelastic Wave Propagation". *Encyclopedia of Thermal Stresses*, 2014. pp. 6462-6467.

[15] D.F.Golla, and P. Hughes, "Dynamics of viscoelastic structures—a time-domain, finite element formulation". *Journal of applied mechanics*, 52(4).1985.pp. 897-906.

[16] J.F.Doyle, "Wave propagation in structures". 1989. Springer.

[17] Zhang, H.H.Z, "Viscoelastic Parameter Identification based Structure-Thermal Analysis of Rubber Bushing". *Global Journal of Researches In Engineering*, 14(3). 2014.

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