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On the Solution of a Rough Interval Bi-Level Multi-Objective Quadratic Programming Problem

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Abstract – **In this paper, a bi-level multi-objective quadratic programming (BLMOQP) problem is considered where some or all of its coefficients in the objective functions are rough intervals. At the first phase of the solution approach and to avoid the complexity of the problem, two BLMOQP problems with interval coefficients will be formulated. One of these problems is a BLMOQP where all of its coefficients are upper approximation of rough intervals and the other problem is a BLMOQP where all of its coefficients are lower approximations of rough intervals. At the second phase, a membership function is constructed to develop a fuzzy model for obtaining the non-inferior solution of the bi-level multi-objective quadratic programming problem. Finally, an illustrative numerical example is given to demonstrate the obtained results.**

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Keywords – **Quadratic Programming, Rough Intervals Programming, Bi-Level Programming.**

I. Introduction

Rough set theory has been proven to be an excellent mathematical tool dealing with vague description of objects. Pawlak has proposed rough set methodology as a new approach in handling classificatory analysis of vague concepts [1]. In this methodology any vague concept is characterized by a pair of precise concepts called the lower and the upper approximations.

The rough interval programming model consists of a set of constructs for representing multi-objective optimization problems. The primary aim in deciding on these constructs is to strike the right balance between constructs that allow for exploitation by a set of optimization algorithms.

Bi-level programming is a powerful and robust technique for solving hierarchical decision making problem. It has been formulated for a problem in which two decision-maker make decisions successively. It has been applied in many real life problems such as economic systems, finance, engineering, banking, management sciences, and transportation problem [2,3,4,5,6].

Recently, notable studies have been done in the area of bi-level multi-objective programming and rough interval programming optimization problems. Also Quadratic Programming is one of the most popular models used in decision-making and in optimization problems [7,8,9,10].

Hamzehee et al. [9] presented a linear programming (LP) problem which is considered where some or all of its coefficients in the objective function and /or constraints are rough intervals. In order to solve this problem, two LP problems with interval coefficients will be constructed. One of these problems is a LP where all of its coefficients are upper approximations of rough intervals and the other problem is a LP where all of its coefficients are lower approximations of rough intervals. Using these two LPs, two newly solutions are defined.

Sultan et al.[10] presented a bi-level linear programming problem with linear constraints, in which the linear objective functions are to be maximized with different rough goals. The suggested approach develops the optimal solution of the bi-level decision- maker, and then the concepts were used of tolerance membership function technique to generate the optimal solution for this problem.

In [4] Osman et al. provided rough bi-level programming problems using genetic algorithm (GA) by constructing the fitness function of the upper level programming problems based on the definition of through feasible degree.

In [11] Osman et al. provided a solution method for solving multi-level non-linear multi-objective under fuzziness. This solution method uses the concepts of tolerance membership functions and multi-objective non-inferior at every level to develop a fuzzy max-min decision model till generating the non-inferior solution.

On the other hand, Emam[3] presented a fuzzy approach for bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level were maximized. It proposed a two planner integer model and a solution method for solving this problem. Therefore Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [7].

The purpose of the present paper is to find a non-inferior solution of the model of a bi-level multi-objective quadratic programming problem with rough interval coefficients. The used fuzzy approach is based mainly upon a systematic access to the best results.

This paper is organized as follows: In Section 2, the bi-level multi-objective quadratic programming problem with rough interval coefficients is formulated. Section 3 involves the theories used to transform rough interval to crisp variable. The fuzzy approach using membership function to solve the problem under consideration is given in Section 4.Section 5 provides an algorithm of finding the non-inferior solution of the formulated model. A numerical example which illustrates the theory of the solution algorithm is suggested in Section 6. Finally, the

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paper is concluded in Section 7 where some points of further research are reported.

II. Problem Formulation and Solution Concept

The bi-level multi-objective quadratic programming problem with rough interval coefficients in the objective functions (BLMOQPRIC) may be written as follows:

[1st Level]

$$\underset{x_1}{Max} F_1(x) = \sum_{j=1}^{n}([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])x_j + \frac{1}{2}x_j^T([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])x_j, \quad (1)$$

where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max} F_2(x) = \sum_{j=1}^{n}([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])x_j + \frac{1}{2}x_j^T([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])x_j, \quad (2)$$

Subject to

$$\mathrm{G} = \{x|\mathrm{A}x \le \mathrm{d}, x \ge 0\}. \quad (3)$$

Where $F_1 = (f_{11},\dots,f_{1n}), F_2 = (f_{21},\dots,f_{2n})$ are the objective functions of the first level decision maker (FLDM), second level decision maker (SLDM), respectively. G is the bi-level multi-objective convex constraint set. Also $([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])$ is rough interval coefficient of the objective function. Let *(j=1,2,…,n)*, $x = (x_1,x_2,\dots,x_n)^T$ denote the vector of all decision variables.

Remark 1.[9]

According to rough interval properties we have:

$$[\underline{c}_j^L,\underline{c}_j^U] \subseteq [\overline{c}_j^L,\overline{c}_j^U] \Rightarrow \overline{c}_j^L \le \underline{c}_j^L \le \underline{c}_j^U \le \overline{c}_j^U.$$

Definition 1.[9]

Consider allofthe corresponding (BLMOQPRIC) problem (1)-(3):

(a) The interval $[\underline{F}^L,\underline{F}^U]\left([\overline{F}^L,\overline{F}^U]\right)$ is called the surely(possibly) non-inferior range of problem (1)-(3), if the non-inferior range of eachBLMOQPRIC Problem is a superset (subset) of $[\underline{F}^L,\underline{F}^U]\left([\overline{F}^L,\overline{F}^U]\right)$.

(b) Let $[\underline{F}^L,\underline{F}^U]\left([\overline{F}^L,\overline{F}^U]\right)$ be surely non-inferior (possibly) non-inferior range of the problem (1)-(3). Then the rough interval $[\underline{F}^L,\underline{F}^U]\left([\overline{F}^L,\overline{F}^U]\right)$is called the roughnon-inferior range of problem (1)-(3).

(c) The non-inferior solution of each corresponding BLMOQPRIC problem (1)-(3) which its non-inferior value belongs to $[\underline{F}^L,\underline{F}^U]\left([\overline{F}^L,\overline{F}^U]\right)$ is called a satisfactory solution of problem (1)-(3).

III. The Transformation of Random Rough Interval Coefficient

The bi-level multi-objective quadratic programming problem with random rough interval coefficient in the objective functions is converted into upper and lower approximations for crisp equivalent in the following manner.

(LI): *Lower interval in the objective functions.*

[1st Level]

$$\underset{x_1}{Max} F_1(x) = \sum_{j=1}^{n}[\underline{c}_j^L,\underline{c}_j^U]x_j + \frac{1}{2}x_j^T[\underline{c}_j^L,\underline{c}_j^U]x_j, \quad (4)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max} F_2(x) = \sum_{j=1}^{n}[\underline{c}_j^L,\underline{c}_j^U]x_j + \frac{1}{2}x_j^T[\underline{c}_j^L,\underline{c}_j^U]x_j, \quad (5)$$

Subject to

$$\mathrm{G} = \{x|\mathrm{A}x \le \mathrm{d}, x \ge 0\}. \quad (6)$$

(UI): *Upper interval in the objective functions.*

[1st Level]

$$\underset{x_1}{Max} F_1(x) = \sum_{j=1}^{n}[\overline{c}_j^L,\overline{c}_j^U]x_j + \frac{1}{2}x_j^T[\overline{c}_j^L,\overline{c}_j^U]x_j, \quad (7)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max} F_2(x) = \sum_{j=1}^{n}[\overline{c}_j^L,\overline{c}_j^U]x_j + \frac{1}{2}x_j^T[\underline{c}_j^L,\underline{c}_j^U]x_j, \quad (8)$$

Subject to

$$\mathrm{G} = \{x|\mathrm{A}x \le \mathrm{d}, x \ge 0\}. \quad (9)$$

Where $i = (1,\dots,k)$, after the division of random rough interval coefficient in the objective functions into upper and lower interval to build a crisp equivalent model, the following theorems are necessary and useful.

Theorem1. [9]

Suppose that the optimal range of **LPIC** problem **(L)** exists. Then, it is equal to the surely optimal range of problem (1)-(3). The optimal range of **LPIC** Problem **(L)** can be obtained by solving two classical **LPs** as follows:

$$P_1: \underline{F}^L := Max \sum_{j=1}^{n} \underline{c}_j^L x_j, \qquad P_2: \underline{F}^U := Max \sum_{j=1}^{n} \underline{c}_j^U x_j,$$

subject to $\sum_{j=1}^{n} \underline{a}_{ij}^U x_j \le \underline{b}_i^L, i = 1,2,\dots,m,$ $x_j \ge 0, j = 1,2,\dots,n,$ subject to $\sum_{j=1}^{n} \underline{a}_{ij}^L x_j \le \underline{b}_i^U, i = 1,2,\dots,m,$ $x_j \ge 0, j = 1,2,\dots,n.$

Theorem2. [9]

Suppose that the optimal range of **LPIC** Problem **(U)** exists. Then, it is equal to the surely optimal range of Problem (1)-(3).The optimal range of **LPIC** Problem **(U)** can be obtained by solving two classical **LPs** as follows:

$$P_3: \overline{F}^L := Max \sum_{j=1}^{n} \overline{c}_j^L x_j, \qquad P_4: \overline{F}^U := Max \sum_{j=1}^{n} \overline{c}_j^U x_j,$$

subject to $\sum_{j=1}^{n} \overline{a}_{ij}^U x_j \le \overline{b}_i^L, i = 1,2,\dots,m,$ $x_j \ge 0, j = 1,2,\dots,n.$ subject to $\sum_{j=1}^{n} \overline{a}_{ij}^L x_j \le \overline{b}_i^U, i = 1,2,\dots,m,$ $x_j \ge 0, j = 1,2,\dots,n.$

Now, the lower interval **LI** and the upper interval **UI**ofproblems given before by (4)-(9) are the reformulated more explicitly as:

a. Lower Interval:

a.1 Lower Interval coefficient (LIC)

[1st Level]

$$\underset{x_1}{Max}\, \underline{f}_{1i}^{L}(x) = \sum_{j=1}^{n} \underline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{L} x_j, \quad (10)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max}\, \underline{f}_{2i}^{L}(x) = \sum_{j=1}^{n} \underline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{L} x_j, \quad (11)$$

Subject to

$$G = \{x | Ax \le d, x \ge 0\}. \quad (12)$$

a.2 Upper Intervalcoefficient (UIC)

[1st Level]

$$\underset{x_1}{Max}\, \underline{f}_{1i}^{U}(x) = \sum_{j=1}^{n} \underline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{U} x_j, \quad (13)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max}\, \underline{f}_{2i}^{U}(x) = \sum_{j=1}^{n} \underline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{U} x_j, \quad (14)$$

Subject to

$$G = \{x | Ax \le d, x \ge 0\}. \quad (15)$$

b. Upper Interval:

b.1 Lower Interval coefficient (LIC)

[1st Level]

$$\underset{x_1}{Max}\, \overline{f}_{1i}^{L}(x) = \sum_{j=1}^{n} \overline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{L} x_j, \quad (16)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max}\, \overline{f}_{2i}^{L}(x) = \sum_{j=1}^{n} \overline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{L} x_j, \quad (17)$$

Subject to

$$G = \{x | Ax \le d, x \ge 0\}. \quad (18)$$

b.2 Upper Interval coefficient (UIC)

[1st Level]

$$\underset{x_1}{Max}\, \overline{f}_{1i}^{U}(x) = \sum_{j=1}^{n} \overline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{U} x_j, \quad (19)$$

Where $\boldsymbol{x_2}$ solves

[2nd Level]

$$\underset{x_2}{Max}\, \overline{f}_{2i}^{U}(x) = \sum_{j=1}^{n} \overline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{U} x_j, \quad (20)$$

Subject to

$$G = \{x | Ax \le d, x \ge 0\}. \quad (21)$$

IV. Fuzzy Approach of Bi-Level Multi-Objective Quadratic Programming Problem [11]

In this section, the bi-level multi-objective quadratic programming problem with rough interval coefficients in the objective functions is solved by using fuzzy approach as described in [11]. At the beginning, we start by stating the first level decision maker problem in the following:

a. First level decision maker problem

The FLDM problem may be formulated as follows:

$$\underset{x_1}{Max}\, F_1(x) = \underset{x_1}{Max}(f_{11}, \dots, f_{1n}), \quad (22)$$

Subject to

$$x \in G. \quad (23)$$

Find individual optimal solution of problem FLDM by obtaining the best and the worst solutions of his problem after transformation (22) problem into the classical problem by theorem (1,2):

$$\underset{x_1}{Max}\, F_1(x):$$

$$\begin{cases} (\boldsymbol{LIC}): \underline{f}_{1i}: \begin{cases} \underset{x_1}{Max}\, \underline{f}_{1i}^{L}(x) = \sum_{j=1}^{n} \underline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{L} x_j \Rightarrow \begin{cases} \left(\underline{f}_{1i}^{L}\right)^{*} = \underset{x\epsilon G}{Max}\, \underline{f}_{1i}^{L}, \\ \left(\underline{f}_{1i}^{L}\right)^{-} = \underset{x\epsilon G}{Min}\, \underline{f}_{1i}^{L}. \end{cases} \\ \underset{x_1}{Max}\, \underline{f}_{1i}^{U}(x) = \sum_{j=1}^{n} \underline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \underline{c}_j^{U} x_j \Rightarrow \begin{cases} \left(\underline{f}_{1i}^{U}\right)^{*} = \underset{x\epsilon G}{Max}\, \underline{f}_{1i}^{U}, \\ \left(\underline{f}_{1i}^{U}\right)^{-} = \underset{x\epsilon G}{Min}\, \underline{f}_{1i}^{U}. \end{cases} \end{cases} \\ (\boldsymbol{UIC}): \overline{f}_{1i}: \begin{cases} \underset{x_1}{Max}\, \underline{f}_{1i}^{L}(x) = \sum_{j=1}^{n} \overline{c}_j^{L} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{L} x_j \Rightarrow \begin{cases} \left(\overline{f}_{1i}^{L}\right)^{*} = \underset{x\epsilon G}{Max}\, \overline{f}_{1i}^{L}, \\ \left(\overline{f}_{1i}^{L}\right)^{-} = \underset{x\epsilon G}{Min}\, \overline{f}_{1i}^{L}. \end{cases} \\ \underset{x_1}{Max}\, \overline{f}_{1i}^{U}(x) = \sum_{j=1}^{n} \overline{c}_j^{U} x_j + \frac{1}{2} x_j^{T} \overline{c}_j^{U} x_j \Rightarrow \begin{cases} \left(\overline{f}_{1i}^{U}\right)^{*} = \underset{x\epsilon G}{Max}\, \overline{f}_{1i}^{U}, \\ \left(\overline{f}_{1i}^{U}\right)^{-} = \underset{x\epsilon G}{Min}\, \overline{f}_{1i}^{U}. \end{cases} \end{cases} \end{cases} \quad (24)$$

Where $i = (1, \dots, k)$, and this data can then be formulated as the following membership function:

$$(\boldsymbol{LIC}): \mu\left[\underline{f}_{1i}(x)\right] =$$

$$\begin{cases} \mu\left[\underline{f}_{1i}^{L}(x)\right] = \begin{cases} 1 & if \quad f_{1i}(x) > \left(\underline{f}_{1i}^{L}\right)^{*}, \\ \dfrac{f_{1i}(x) - \left(\underline{f}_{1i}^{L}\right)^{-}}{\left(\underline{f}_{1i}^{L}\right)^{*} - \left(\underline{f}_{1i}^{L}\right)^{-}} & if \quad \left(\underline{f}_{1i}^{L}\right)^{-} \le f_{1i}(x) \le \left(\underline{f}_{1i}^{L}\right)^{*}, \\ 0 & if \quad \left(\underline{f}_{1i}^{L}\right)^{-} \ge f_{i1}(x). \end{cases} \\ \mu\left[\underline{f}_{1i}^{U}(x)\right] = \begin{cases} 1 & if \quad f_{1i}(x) > \left(\underline{f}_{1i}^{U}\right)^{*}, \\ \dfrac{f_{1i}(x) - \left(\underline{f}_{1i}^{U}\right)^{-}}{\left(\underline{f}_{1i}^{U}\right)^{*} - \left(\underline{f}_{1i}^{U}\right)^{-}} & if \quad \left(\underline{f}_{1i}^{U}\right)^{-} \le f_{1i}(x) \le \left(\underline{f}_{1i}^{U}\right)^{*}, \\ 0 & if \quad \left(\underline{f}_{1i}^{U}\right)^{-} \ge f_{1i}(x). \end{cases} \end{cases} \quad (25)$$

$$(\boldsymbol{UIC}): \boldsymbol{\mu}\left[\overline{\boldsymbol{f}}_{\boldsymbol{1i}}(\boldsymbol{x})\right] =$$

$$\begin{cases} \mu\left[\overline{f}_{1i}^{L}(x)\right] = \begin{cases} 1 & if \quad f_{1i}(x) > \left(\overline{f}_{1i}^{L}\right)^{*}, \\ \dfrac{f_{1i}(x) - \left(\overline{f}_{1i}^{L}\right)^{-}}{\left(\overline{f}_{1i}^{U}\right)^{*} - \left(\overline{f}_{1i}^{L}\right)^{-}} & if \quad \left(\overline{f}_{1i}^{L}\right)^{-} \le f_{1i}(x) \le \left(\overline{f}_{1i}^{L}\right)^{*}, \\ 0 & if \quad \left(\overline{f}_{1i}^{L}\right)^{-} \le f_{1i}(x). \end{cases} \\ \mu\left[\overline{f}_{1i}^{U}(x)\right] = \begin{cases} 1 & if \quad f_{1i}(x) > \left(\overline{f}_{1i}^{L}\right)^{*}, \\ \dfrac{f_{1i}(x) - \left(\overline{f}_{1i}^{U}\right)^{-}}{\left(\overline{f}_{1i}^{U}\right)^{*} - \left(\overline{f}_{1i}^{U}\right)^{-}} & if \quad \left(\overline{f}_{1i}^{L}\right)^{-} \le f_{1i}(x) \le \left(\overline{f}_{1i}^{L}\right)^{*}, \\ 0 & if \quad \left(\overline{f}_{1i}^{L}\right)^{-} \le f_{1i}(x). \end{cases} \end{cases} \quad (26)$$

Now, the solution of the FLDM problem can be obtained by solving the following Tchebycheff problem:

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$$(\boldsymbol{LIC}):\begin{cases} Max\lambda, \\ subject\ to \\ x\in G, \\ \mu\left[\underline{f}_{1i}^{L}(x)\right]\geq\lambda, \\ \lambda\in[0,1]. \\ \\ Max\lambda, \\ subject\ to \\ x\in G, \\ \mu\left[\underline{f}_{1i}^{U}(x)\right]\geq\lambda, \\ \lambda\in[0,1]. \end{cases} \qquad (27)$$

$$(\boldsymbol{UIC}):\begin{cases} Max\lambda, \\ subject\ to \\ x\in G, \\ \mu\left[\overline{f}_{1i}^{L}(x)\right]\geq\lambda, \\ \lambda\in[0,1]. \\ \\ Max\lambda, \\ subject\ to \\ x\in G, \\ \mu\left[\overline{f}_{1i}^{U}(x)\right]\geq\lambda, \\ \lambda\in[0,1]. \end{cases} \qquad (28)$$

Whose solution are assumed to be $(\underline{x}_1^L,\underline{x}_2^L,\underline{x}_3^L)^F$, $(\underline{x}_1^U,\underline{x}_2^U,\underline{x}_3^U)^F$, $(\overline{x}_1^L,\overline{x}_2^L,\overline{x}_3^L)^F$, $(\overline{x}_1^U,\overline{x}_2^U,\overline{x}_3^U)^F$, λ^F, and $\left([\underline{f}_{1i}^L,\underline{f}_{1i}^U],[\overline{f}_{1i}^L,\overline{f}_{1i}^U]\right)^F$, (satisfactory level).

b. *Second level decision maker problem*

The SLDM does the same action like the FLDM till he/she obtains his/her solution to be $(\underline{x}_1^L,\underline{x}_2^L,\underline{x}_3^L)^S$, $(\underline{x}_1^U,\underline{x}_2^U,\underline{x}_3^U)^S$, $(\overline{x}_1^L,\overline{x}_2^L,\overline{x}_3^L)^S$, $(\overline{x}_1^U,\overline{x}_2^U,\overline{x}_3^U)^S$, β^S, and $\left([\underline{f}_{2i}^L,\underline{f}_{2i}^U],[\overline{f}_{2i}^L,\overline{f}_{2i}^U]\right)^S$, (satisfactory level).

Now, the solution of the FLDM, and SLDM, are disclosed. However, two solutions are usually different because of nature between two levels multi-objective functions.

The FLDM knows that using the optimal decisions $x_1^F:\{(\underline{x}_1^L)^F,(\underline{x}_1^U)^F,(\overline{x}_1^L)^F,(\overline{x}_1^U)^F\}$,as a control factors for the SLDM, are not practical. It is more reasonable to have some tolerance that gives the SLDM an extent feasible region to search for his/her non-inferior solution, and reduce searching time or interactions.

In this way, the range of decision variable $x_1:\{\underline{x}_1^L,\underline{x}_1^U,\overline{x}_1^L,\overline{x}_1^U\}$, should be around $x_1^F:\{(\underline{x}_1^L)^F,(\underline{x}_1^U)^F,(\overline{x}_1^L)^F,(\overline{x}_1^U)^F\}$ with maximum tolerance t and the following membership function specify $\left\{(\underline{x}_1^L)^F,(\underline{x}_1^U)^F,(\overline{x}_1^L)^F,(\overline{x}_1^U)^F\right\}$as:

$$\mu(x_1):\begin{cases} \mu(\underline{x}_1)=\begin{cases} \mu(\underline{x}_1^L)=\begin{cases} \dfrac{\underline{x}_1-\left((\underline{x}_1^L)^F-t\right)}{t} & if\ \ (\underline{x}_1^L)^F-t\leq\underline{x}_1\leq(\underline{x}_1^L)^F \\ \dfrac{-\underline{x}_1+\left((\underline{x}_1^L)^F+t\right)}{t} & if\ \ (\underline{x}_1^L)^F\leq\underline{x}_1\leq(\underline{x}_1^L)^F-t \end{cases} \\ \mu(\underline{x}_1^U)=\begin{cases} \dfrac{\underline{x}_1-\left((\underline{x}_1^U)^F-t\right)}{t} & if\ \ (\underline{x}_1^U)^F-t\leq\underline{x}_1\leq(\underline{x}_1^U)^F \\ \dfrac{-\underline{x}_1+\left((\underline{x}_1^U)^F+t\right)}{t} & if\ \ (\underline{x}_1^U)^F\leq\underline{x}_1\leq(\underline{x}_1^U)^F-t \end{cases} \end{cases} \\ \mu(\overline{x}_1)=\begin{cases} \mu(\overline{x}_1^L)=\begin{cases} \dfrac{\overline{x}_1-\left((\overline{x}_1^L)^F-t\right)}{t} & if\ \ (\overline{x}_1^L)^F-t\leq\overline{x}_1\leq(\overline{x}_1^L)^F \\ \dfrac{-\overline{x}_1+\left((\overline{x}_1^L)^F+t\right)}{t} & if\ \ (\overline{x}_1^L)^F\leq\overline{x}_1\leq(\overline{x}_1^L)^F-t \end{cases} \\ \mu(\overline{x}_1^U)=\begin{cases} \dfrac{\overline{x}_1-\left((\overline{x}_1^U)^F-t\right)}{t} & if\ \ (\overline{x}_1^U)^F-t\leq\overline{x}_1\leq(\overline{x}_1^U)^F \\ \dfrac{-\overline{x}_1+\left((\overline{x}_1^U)^F+t\right)}{t} & if\ \ (\overline{x}_1^U)^F\leq\overline{x}_1\leq(\overline{x}_1^U)^F-t \end{cases} \end{cases} \end{cases} \qquad (29)$$

The FLDM goals may reasonably consider $F_1\left(\bar{f}_1^L,\bar{f}_1^U,\underline{f}_1^L,\underline{f}_1^U\right)\geq F_1{}^F((\underline{f}_1^L)^F,(\underline{f}_1^U)^F,(\overline{f}_1^L)^F,(\overline{f}_1^U)^F)$ is absolutely acceptable and

$$F_1\leq F_1^F(x_1^S,x_2^S,x_3^S):\begin{pmatrix} \left\{(\underline{x}_1^L)^S,(\underline{x}_1^U)^S,(\overline{x}_1^L)^S,(\overline{x}_1^U)^S\right\} \\ ,\left\{(\underline{x}_2^L)^S,(\underline{x}_2^U)^S,(\overline{x}_2^L)^S,(\overline{x}_2^U)^S\right\} \\ ,\left\{(\underline{x}_3^L)^S,(\underline{x}_3^U)^S,(\overline{x}_3^L)^S,(\overline{x}_3^U)^S\right\} \end{pmatrix}$$ is

absolutely unacceptable, and that the preference with $[\acute{F}_1,F_1^F]$ is linearly increasing. This is due to the fact that the SLDM obtained the optimum

at$(x_1^S,x_2^S,x_3^S):\begin{pmatrix} \left\{(\underline{x}_1^L)^S,(\underline{x}_1^U)^S,(\overline{x}_1^L)^S,(\overline{x}_1^U)^S\right\} \\ ,\left\{(\underline{x}_2^L)^S,(\underline{x}_2^U)^S,(\overline{x}_2^L)^S,(\overline{x}_2^U)^S\right\} \\ ,\left\{(\underline{x}_3^L)^S,(\underline{x}_3^U)^S,(\overline{x}_3^L)^S,(\overline{x}_3^U)^S\right\} \end{pmatrix}$,which in turn provides the FLDM the objective function values $\acute{F}_1$, makes any $F_1\geq\acute{F}_1=F_1(x_1^S,x_2^S,x_3^S)$ unattractive in practice.

The following membership functions of the FLDM can be stated as:

$$\acute{\mu}[F_1(x)]=\begin{cases} 1 & if & F_1(x)>F_1^F, \\ \dfrac{F_1(x)-\acute{F}_1}{F_1^F-\acute{F}_1} & if & \acute{F}_1\leq F_1(x)\leq F_1^F, \\ 0 & if & \acute{F}_1\geq F_1(x). \end{cases} \qquad (30)$$

Second, The SLDM do the same action like the FLDM.

Finally, in order to generate the satisfactory solution, which is also non-inferior solution with overall satisfaction for all decision-makers, the following Tchebycheff problem will be solved:

$$Max\ \theta, \qquad (31)$$

Subject to

$$\begin{aligned} &\acute{\mu}[F_1(x)]\geq\theta, \\ &\acute{\mu}[F_2(x)]\geq\theta, \\ &\frac{[x_1-(x_1^F-t)]}{t}\geq\theta, \\ &\frac{-x_1+(x_1^F+t)}{t}\geq\theta, \\ &x\in G, \\ &t>0, \\ &\theta\in[0,1]. \end{aligned}$$

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V. AN ALGORITHM FOR SOLVING PROBLEM (BLMOQPRIC)

A solution algorithm to solve (BLMOQPRIC) problems (1)-(3)is described in a series of steps as follows:

Step 1: Determine the possible random rough inter valco efficient rang (lower (L) interval problem) in FLDM and SLDM problem, respectively.

Step 2: Determine the rough interval coefficient rang (upper (U) interval problem) in FLDM, and SLDM problem, go to step3.

Step 3: Formulate the corresponding equivalent problem (BLMOQPP).

Step 4: Convert the lower and upper random interval coefficient in FLDM problem in to equivalent crisp models can be solved by classical methods.

Step 5: Convert the lower and upper random interval coefficient SLDM problem in to equivalent crisp models, go to step 6.

Step 6: Using the fuzzy approach as described in [11] to solve the resulting multi-level multi-objective decision-making problems in Step 5.

Step 7: Build membership functions of the FLDM, and SLDM after determine the best and the worst solution of all (LIC) and (UIC) problems.

Step 8: Solve a Tchebycheff problem for all decision makers level problem.

Step 9: Control assumed the FLDM his /her decision by tolerancet_1.

*Step 10:*If$\beta < 0$,increase t then go to step7, otherwise go to step 12.

Step 11: The FLDM and SLDM calculating membership functionµ̀.

Step 12: Compute tolerance functions for $x_1$using t.

Step 13: Solve the Tchebycheff problem defined by (31), then go to step15.

Step 14: If the FLDM not satisfied with solution then go to step 9 with modifying, $\theta(\underline{\theta}^L, \underline{\theta}^U, \overline{\theta}^L, \overline{\theta}^U)$.

Step15: Stop.

VI. NUMERICAL EXAMPL

To demonstrate the solution method for bi-level multi-objective interval quadratic programming problem under random rough coefficient in objective functions, let us consider the following example:

[1stLevel]

$$\underset{x_1}{Max}\, F_1(x) = \begin{pmatrix} 2([2,3],[1,5])x_1 + 3([0,3],[0,4])x_2 + 8x_3^2, \\ 6x_1 + 4([2,3],[0,4])x_2^2 + 2([3,4],[1,5])x_3 \end{pmatrix},$$

wherex_2 solves

[2^{sd}Level]

$$\underset{x_2}{Max}\, F_2(x)$$

$$= \begin{pmatrix} 2x_1^2 + 12x_2 + 4([1,2],[0,3])x_2^2 + 5([1,3],[0,4])x_3^2, \\ 10([0,3],[0,4])x_1 + 9x_1^2 + 5x_2^2 + 3([1,2],[1,5])x_3 \end{pmatrix},$$

Subject to

$3x_1 + 5x_2 + x_3 \le 35,$
$2x_1 - x_2 + 12x_3 \le 20,$
$5x_2 + 6x_3 \le 16\,,$
$x_i \ge 0, i = 1,2,3.$

Where;

$\boldsymbol{f_{11}} = 2([2,3],[1,5])x_1 + 3([0,3],[0,4])x_2 + 8x_3^2,$
$\boldsymbol{f_{12}} = 6x_1 + 4([2,3],[0,4])x_2^2 + 2([3,4],[1,5])x_3,$
$\boldsymbol{f_{21}} = 2x_1^2 + 12x_2 + 4([1,2],[0,3])x_2^2 + 5([1,3],[0,4])x_3^2,$
$\boldsymbol{f_{22}} = 10([0,3],[0,4])x_1 + 9x_1^2 + 5x_2^2 + 3([1,2],[1,5])x_3.$

Now by using Theorem (1,2), the equivalent crisp problems which equivalent to bi-level multi- objective interval quadratic programming problem under random rough coefficient in objective functions, can be written as:

1stLevel	
Lower	
QP1:	**QP2**:
$f_{11}^L \coloneqq Max\ 4x_1 + 8x_3^2,$	$f_{11}^U \coloneqq Max\ 6x_1 + 9x_2 + 8x_3^2,$
$f_{12}^L \coloneqq Max\ 6x_1 + 8x_2^2 + 6x_3,$	$f_{12}^U \coloneqq Max\ 6x_1 + 12x_2^2 + 8x_3,$
Subject to	**Subject to**
$3x_1 + 5x_2 + x_3 \le 35,$	$3x_1 + 5x_2 + x_3 \le 35,$
$2x_1 - x_2 + 12x_3 \le 20,$	$2x_1 - x_2 + 12x_3 \le 20,$
$5x_2 + 6x_3 \le 16,$	$5x_2 + 6x_3 \le 16,$
$x_i \ge 0, i = 1,2,3.$	$x_i \ge 0, i = 1,2,3.$
Upper	
QP3:	**QP4**:
$\overline{f}_{11}^L \coloneqq Max\, 2x_1 + 8x_3^2,$	$\overline{f}_{11}^U \coloneqq Max\, 10x_1 + 12x_2$ $+8x_3^2,$
$\overline{f}_{12}^L \coloneqq Max\, 6x_1 + 2x_3,$	$\overline{f}_{12}^U \coloneqq Max\ 6x_1 + 16x_2^2 + 10x_3,$
Subject to	**Subject to**
$3x_1 + 5x_2 + x_3 \le 35,$	$3x_1 + 5x_2 + x_3 \le 35,$
$2x_1 - x_2 + 12x_3 \le 20,$	$2x_1 - x_2 + 12x_3 \le 20,$
$5x_2 + 6x_3 \le 16,$	$5x_2 + 6x_3 \le 16,$
$x_i \ge 0, i = 1,2,3.$	$x_i \ge 0, i = 1,2,3.$

Table (1.a)

2ndLevel	
Lower	
QP1:	**QP2:**
$f_{21}^L \coloneqq Max\ 2x_1^2 + 12x_2$ $+4x_2^2 + 5x_3^2,$	$f_{21}^U \coloneqq Max\ 2x_1^2 + 12x_2 + 8x_2^2$ $+15x_3^2,$
$f_{22}^L \coloneqq Max\ 9x_1^2 + 5x_2^2$ $+3x_3,$	$f_{22}^U \coloneqq Max\ 30x_1 + 9x_1^2 + 5x_2^2$ $+6x_3,$
Subject to	**Subject to**
$3x_1 + 5x_2 + x_3 \le 35,$	$3x_1 + 5x_2 + x_3 \le 35,$
$2x_1 - x_2 + 12x_3 \le 20,$	$2x_1 - x_2 + 12x_3 \le 20,$
$5x_2 + 6x_3 \le 16,$	$5x_2 + 6x_3 \le 16,$
$x_i \ge 0, i = 1,2,3.$	$x_i \ge 0, i = 1,2,3.$
Upper	
QP3:	**QP4:**
$\overline{f}_{21}^L \coloneqq Max\ 2x_1^2 + 12x_2, \overline{f}_{22}^L \coloneqq Max\, 9x_1^2 + 5x_2^2 + 3x_3,$	$\overline{f}_{21}^U \coloneqq Max\ 2x_1^2 + 12x_2 + 12x_2^2 + 20x_3^2,$
	$\overline{f}_{22}^U \coloneqq Max\ 40x_1 + 9x_1^2$ $+5x_2^2 + 15x_3,$

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Subject to	**Subject to**
$3x_1 + 5x_2 + x_3 \le 35,$	$3x_1 + 5x_2 + x_3 \le 35,$
$2x_1 - x_2 + 12x_3 \le 20,$	$2x_1 - x_2 + 12x_3 \le 20,$
$5x_2 + 6x_3 \le 16,$	$5x_2 + 6x_3 \le 16,$
$x_i \ge 0, i = 1,2,3.$	$x_i \ge 0, i = 1,2,3.$

Table (2.b)

By using (25,26), the FLDM build the membership functions $\mu\left(f_{11}^L, f_{11}^U, \bar{f}_{11}^L, \bar{f}_{11}^U\right)(x)$, $\mu(f_{12}^L, f_{12}^U, \bar{f}_{12}^L, \bar{f}_{12}^U)(x)$ and from table (1.a) then solve problem as follows:

QP1:

$$Max\ \lambda,$$

subject to

$4x_1 + 8x_3^2 \le 41.53846\lambda$
$6x_1 + 8x_2^2 + 6x_3 \le 119.9200\lambda,$
$x \in G\,,$
$\lambda \in [0,1].$
Whose solutionis
$(\underline{x}_1^L, \underline{x}_2^L, \underline{x}_3^L)^F = (1.234568, 1.234568, 1.234568)$
$(\underline{f}_{11}^L)^F$**=17.13153717,**$(\underline{f}_{12}^L)^F$**=27.0080811,**$\lambda^F = \mathbf{0.9}$.

QP2:
$Max\ \lambda,$
subject to
$6x_1 + 9x_2 + 8x_3^2 \le 69.53746\lambda,$
$6x_1 + 12x_2^2 + 8x_3 \le 160.8800\lambda,$
$x \in G\,,$
$\lambda \in [0,1].$
Whose solution is
$(\underline{x}_1^U, \underline{x}_2^U, \underline{x}_3^U)^F = (1.234568, 1.234568, 1.234568)$
$(\underline{f}_{11}^U)^F$**=30.71178517,**$(\underline{f}_{12}^U)^F$**=35.57384976,** $\lambda^F = \mathbf{0.9}$.

QP3:
$Max\ \lambda,$
subject to
$2x_1 + 8x_3^2 \le 24.71258\lambda,$
$6x_1 + 2x_3 \le 62.30769\lambda,$
$x \in G\,,$
$\lambda \in [0,1].$
Whose solution is
$\left(\bar{x}_1^L, \bar{x}_2^L, \bar{x}_3^L\right)^F = (1.234568, 1.234568, 1.234568)$
$(\bar{f}_{11}^L)^F$**=14.66240117,**$(\bar{f}_{12}^L)^F$**=9.876544,**$\lambda^F = \mathbf{0.9}$.

QP4:
$Max\ \lambda,$
subject to
$10x_1 + 12x_2 + 8x_3^2 \le 113.0769\lambda,$
$6x_1 + 16x_2^2 + 10x_3 \le 201.8400\lambda,$
$x \in G\,,$
$\lambda \in [0,1].$
Whose solution is
$(\bar{x}_1^U, \bar{x}_2^U, \bar{x}_3^U)^F = (1.234568, 1.234568, 1.234568)$
$(\bar{f}_{11}^U)^F$**=39.35376117,**$(\bar{f}_{12}^U)^F$**=44.13961835,**$\lambda^F = \mathbf{0.9}$.

Table (1.2.a)

Secondly, the SLDM defines his/her problem in view of the FLDM as follows:

QP1:$\left(\underline{x}_1^L, \underline{x}_2^L, \underline{x}_3^L\right)^S = (1.234568, 1.234568, 1.234568),$
QP2:$(\underline{x}_1^U, \underline{x}_2^U, \underline{x}_3^U)^S = (1.234568, 1.234568, 1.234568).$
QP3:$(\bar{x}_1^L, \bar{x}_2^L, \bar{x}_3^L)^S = (1.234568, 1.234568, 1.234568),$
QP4:$(\bar{x}_1^U, \bar{x}_2^U, \bar{x}_3^U)^S = (1.234568, 1.234568, 1.234568).$

Finally, in order to generate the satisfactory solution, which is also non-inferiorwith overall satisfaction for all decision-makers, by (44), calculating the tolerance function also.

1- We assume the FLDM'S control decision x_1^F with the tolerance 1.
2- By using (29)–(30) calculating membership functions µ̀, then solves the Tchebycheff problem as follows :

$Max\ \theta_1,$
Subject to
$4x_1 + 8x_3^2 - 17.13153717 \ge 0,$
$6x_1 + 8x_2^2 + 6x_3 - 27.0080817 \ge 0,$
$2x_1^2 + 12x_2 + 4x_2^2 + 5x_3^2 - 31.58055561 \ge 0,$
$9x_1^2 + 5x_2^2 + 3x_3 - 25.04191805 \ge 0,$
$x_1 - 0.234568 \ge \theta_1,$
$-x_1 + 2.234568 \ge \theta_1,$
$x \in G,$
$t > 0\,,$
$\theta_1 \in [0,1].$
$(\underline{x}_1^L, \underline{x}_2^L, \underline{x}_3^L) = (1.234568, 1.234568, 1.234568).$

$Max\ \theta_2,$
Subject to
$6x_1 + 9x_2 + 8x_3^2 - 30.71178517 \ge 0,$
$6x_1 + 12x_2^2 + 8x_3 - 35.57384976 \ge 0,$
$2x_1^2 + 12x_2 + 8x_2^2 + 15x_3^2 - 52.91876967 \ge 0,$
$30x_1 + 9x_1^2 + 5x_2^2 + 6x_3 - 65.78266205 \ge 0,$
$x_1 - 0.234568 \ge \theta_2,$
$-x_1 + 2.234568 \ge \theta_2,$
$x \in G,$
$t > 0\,,$
$\theta_2 \in [0,1],$
$(\underline{x}_1^U, \underline{x}_2^U, \underline{x}_3^U) = (1.234568, 1.234568, 1.234568).$

$Max\ \theta_3,$
Subject to
$2x_1 + 8x_3^2 - 14.66240117 \ge 0,$
$6x_1 + 2x_3 - 9.876544 \ge 0,$
$2x_1^2 + 12x_2 - 17.86313229 \ge 0,$
$9x_1^2 + 5x_2^2 + 3x_3 - 25.04191805 \ge 0,$
$x_1 - 0.234568 \ge \theta_3,$
$-x_1 + 2.234568 \ge \theta_3,$
$x \in G,$
$t > 0\,,$
$\theta_3 \in [0,1].$
$(\bar{x}_1^L, \bar{x}_2^L, \bar{x}_3^L) = (1.234568, 1.234568, 1.234568).$

$Max\ \theta_4,$
Subject to
$10x_1 + 12x_2 + 8x_3^2 - 39.35376117 \ge 0,$
$6x_1 + 16x_2^2 + 10x_3 - 44.13961835 \ge 0,$
$2x_1^2 + 12x_2 + 12x_2^2 + 20x_3^2 - 66.63619299 \ge 0,$
$40x_1 + 9x_1^2 + 5x_2^2 + 15x_3 - 89.23945405 \ge 0,$
$x_1 - 0.234568 \ge \theta_4,$
$-x_1 + 2.234568 \ge \theta_4,$
$x \in G,$
$t > 0\,,$
$\theta_4 \in [0,1],$
$(\bar{x}_1^U, \bar{x}_2^U, \bar{x}_3^U) = (1.234568, 1.235685, 1.245730).$

Overall satisfaction for both decisions-makers are:

$\left(\left[\underline{f}_{11}^{L},\underline{f}_{11}^{U}\right],\left[\overline{f}_{11}^{L},\overline{f}_{11}^{U}\right]\right)=$ ([17.13153717,30.71178517],[14.66240117,39.58864586]).

$\left(\left[\underline{f}_{12}^{L},\underline{f}_{12}^{U}\right],\left[\overline{f}_{12}^{L},\overline{f}_{12}^{U}\right]\right)=$ ([27.00808117,35.57384976],[9.876544,44.29538671]).

$\left(\left[\underline{f}_{21}^{L},\underline{f}_{21}^{U}\right],\left[\overline{f}_{21}^{L},\overline{f}_{21}^{U}\right]\right)$ = ([31.58055561,52.91876967],[17.86313229,67.23640998]).

$\left(\left[\underline{f}_{22}^{L},\underline{f}_{22}^{U}\right],\left[\overline{f}_{22}^{L},\overline{f}_{22}^{U}\right]\right)$ =([25.04191805,65.78266205],[25.04191805,89.42068042).

VII. Summary and Concluding Remarks

A bi-level multi-objective quadratic programming (BLMOQP) problem was considered where some or all of its coefficients in the objective function are rough intervals. At the first phase of the solution approach and to avoid the complexity of the problem, two BLMOQP problems with interval coefficients will be formulated. One of these problems was a BLMOQP where all of its coefficients are upper approximation of rough intervals and the other problem was a BLMOQP where all of its coefficients are lower approximations of rough intervals. At the second phase, a membership function was constructed to develop a fuzzy model for obtaining the non-inferior solution of the bi-level multi-objective quadratic programming problem. In addition, the author put forward the satisfactoriness concept as the first-level decision-maker preference.

However, there are many open points for discussion in future, which should be explored and studied in the area of multi- level rough Interval optimization such as:

1. Interactive algorithm is required for treating bi-level multi-objective integer quadratic decision-making problems with rough parameters in the objective functions; in the constraints and in both.

2. Interactive algorithm is needed for dealing with bi-level multi-objective pure integer quadratic decision-making problems with rough parameters in the objective functions; in the constraints and in both.

3. Interactive algorithm is necessary for solving bi-level multi-objective integer fractional decision-making problems with rough parameters in the objective functions; in the constraints and in both.

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